

Generalized Rotemberg Price-Setting

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Abstract

We propose a generalized Rotemberg pricing scheme which is able to explain important aspects of the observed nonlinear behavior of price adjustment at the macroeconomic level, such as higher pass-through in response to larger shocks, a positive impact of trend inflation on price flexibility, and the relationship between announcement and implementation effects. This is achieved by replacing the linear marginal adjustment cost in Rotemberg pricing by a monotonically increasing, but bounded marginal cost function, specifically some version of a sigmoid function. Conditional on computing a nonlinear model solution, the generalized pricing function is equally tractable as Rotemberg pricing, and equivalent to it for small shocks around a zero-inflation steady state. We show that a suitable calibration of the model has similar effects on macroeconomic variables as standard versions of the menu cost model. It replicates the effect of trend inflation on the impulse response to money supply shocks that has been established in the literature in a model of logit-price dynamics.

Keywords: Price setting, nominal rigidities, inflation, DSGE.

JEL Classification Numbers: E13, E31.

1 Introduction

In recent years, significant progress has been made in explaining microfacts of price-setting behavior, using models of heterogeneous firms with often rather complex state-dependent pricing behavior. Menu costs of price adjustment are a typical ingredient of these models. In contrast, medium-scale DSGE models either assume time-dependent staggered pricing a la Calvo (1983), where firm heterogeneity can be analytically aggregated, or assume a representative firm, following Rotemberg (1982). These approaches are much more tractable and provide good approximations to the aggregate responses to small shocks around a zero-inflation steady state. However, it is well known that they fail to capture the observed nonlinear aspects of pricing behavior.

The main contribution of our paper is to propose a generalization of the Rotemberg pricing model (abbreviated GRP in the following) that is equally tractable as the Rotemberg model, but is able to capture the observed nonlinear features of inflation dynamics, which are relevant in the presence of large shocks or positive trend inflation. The generalization consists in replacing the linear marginal cost function by a symmetric sigmoid function, where marginal pricing costs are monotonically increasing but bounded. The model is equivalent to the Rotemberg model up to second-order perturbations around a zero-inflation steady state. To analyze the model, it is therefore necessary to use nonlinear solution techniques such as nonlinear perfect foresight paths, higher-order perturbation solutions or global nonlinear models. In our examples, we compare impulse responses to large shocks obtained from higher-order perturbation and perfect foresight solutions.

What are the salient nonlinear features of inflation dynamics? The literature has established three stylized facts which are explained by some version of state-dependent models but are inconsistent with the simple pricing models of Calvo and Rotemberg. First, Ascari and Haber (2022) document the nonlinear pass-through of cost shocks to prices. Alvarez et al. (2017) point out that this is a key implication of state-dependent pricing models: large shocks propagate faster than small shocks, because the larger the shock, the more firms should find it profitable to adjust their prices. Cavallo et al. (2023) highlight the importance of this mechanism in explaining the inflation surge observed in 2022, as their data show a notable increase in the frequency of price changes following large shocks. They explain this using a New Keynesian model incorporating state-dependent price-setting. Second, a growing body of empirical evidence suggests that higher trend inflation is associated with a steeper New Keynesian Phillips curve (Benati, 2007; Ball and Mazumder, 2011; Gemma et al., 2023). Already Ball et al. (1988) argued that a higher trend inflation rate implies smaller real effects of nominal shocks. Their model illustrates how higher trend inflation induces firms to adjust prices more frequently, thereby mitigating the effects of nominal shocks. Similar mechanisms are present in recent state-dependent price-setting models such as Costain and Nakov

(2019), which implies that higher trend inflation amplifies the impact of monetary policy shocks on inflation while dampening their effect on consumption. A second stylized fact is therefore that higher trend inflation leads to greater aggregate price flexibility. Third, the announcement effects of VAT reforms are comparatively small and the pass-through occurs mainly during implementation, as several recent empirical papers document (Benzarti et al., 2020; Benedek et al., 2020). One possible reason may be that a large share of fixed menu costs in total price adjustment costs leads to a lower incentive to smooth price adjustments over time (Karadi and Reiff, 2019). Another reason may be that firms find it easier to justify price adjustments to their customers during and after the implementation of the VAT reform.

We analyze the properties of the model in three directions. First we investigate to what extent the model can replicate the aggregate implications of state-dependent pricing models. Despite being very parsimoniously parameterized, we show that the model has similar inflation dynamics as standard forms of menu cost models, if properly calibrated. Both models explain the observed nonlinear pass-through of cost shocks to prices and the greater aggregate price flexibility under positive trend inflation. Second, we embed the generalized Rotemberg model into a basic DSGE framework, imposing a parameter restriction such that the model is equivalent up to the standard Calvo or Rotemberg model for small shocks around a zero-inflation steady state. We show that the generalized Rotemberg pricing model goes a long way in explaining the above mentioned stylized facts, in contrast to the standard Rotemberg or the Calvo model. Third, we check whether the model can reproduce the relationship between trend inflation and price flexibility in the case of high rates of trend inflation. We show that our model has similar aggregate implications as the model of logit pricing in Costain and Nakov (2019), which is successful in explaining the firm level evidence for Mexican data with trend inflation rates of up to 80 percent annually.

Our approach builds on Rotemberg (1982), who developed a theory proposing that firms, concerned about upsetting their customers, attribute quadratic costs to price changes. Zbaracki et al. (2004) provided detailed microeconomic evidence that supports such a convex relationship. Their research suggests that menu costs, as traditionally understood, constitute only a small portion of the overall costs associated with setting prices. Instead, the study highlights the significance of managerial costs, such as information gathering, decision-making, and communication costs, as well as customer costs like communication and negotiation expenses. Importantly, Zbaracki et al. (2004) demonstrate that many components of these managerial and customer costs are convex in the size of the price adjustment. Stipulating quadratic costs, as Rotemberg originally did, is overly restrictive.¹ Our approach improves on this by allowing for a more general and more plausible class of convex functions. The literature also provides empirical support

¹Rotemberg refined his arguments in later work (Rotemberg, 2005, 2010, 2011).

for Rotemberg pricing, suggesting that it is empirically not inferior to the Calvo model, cf. Section 2.2 below. We build on the Rotemberg rather than the Calvo model, because it allows a nonlinear generalization in a straightforward and tractable way.

The paper proceeds as follows. After the literature review in Section 2, Section 3 introduces the generalization of Rotemberg pricing by means of a sigmoid marginal cost function. In section 4 we use a partial equilibrium industry model to compare the pass-through in the generalised Rotemberg model with that in a menu cost model. Section 5 embeds generalized Rotemberg pricing into a simple New Keynesian DSGE model, to explain the aforementioned stylized facts. The case study in Section 6 demonstrates the ability of the generalized Rotemberg model to reproduce the implications of trend inflation in the model of Costain and Nakov (2019). Section 7 concludes.

2 Related Literature

2.1 Models of price setting

There is now an extensive literature devoted to models in which price rigidity is based on menu costs. An important topic of this literature is whether and how menu costs at the micro level lead to monetary non-neutrality in the aggregate. The former has been questioned by Caplin and Spulber (1987) and Golosov and Lucas (2007), the latter has been investigated by e.g. Caballero and Engel (2007), who study the importance of selection effects versus adjustments to the extensive margin, and Nakamura and Steinsson (2010), who include additional model components and in this way increase monetary non-neutrality.

Recently, the literature has generalized these models, making it possible, for example, to derive sufficient statistics for the effects of monetary policy (Alvarez et al. (2016)). In addition, Alvarez et al. (2022) have shown that in a large class of sticky price models the price-setting behavior of firms can be described by a generalized hazard function, and Auclert et al. (2024) have investigated to what extent the Calvo model can approximate menu costs models with appropriate calibration. We will briefly discuss in section 3.3 how this literature can be used to calibrate the generalized Rotemberg model.

As data availability has improved at the micro level, a large literature has emerged that attempts to explain the detailed stylized facts reported by, for example, Klenow and Malin (2010), by constructing increasingly complex models of price setting. Nakamura and Steinsson (2013) provided an early review of this literature, important recent contributions have been made by Costain and Nakov (2019), Karadi and Reiff (2019), Ilut et al. (2020) and Dotsey and Wolman (2020). These are complex state-dependent models of price-setting, in which price adjustments occur at both the intensive and the extensive margin, in which possible selection effects are taken into account and which may or may

not be based on menu costs. In Section 6, we show that our generalized Rotemberg model is able to reproduce the behavior of Costain and Nakov’s model in the presence of high trend inflation, a model in which price rigidities are not based on menu costs but on error-prone decision making.

While Calvo’s (1983) model has been used more frequently in the meantime, the Rotemberg model has recently become much more popular, especially due to the Heterogeneous Agents New Keynesian (HANK) literature. Indeed, it is used in important milestones of this literature such as Kaplan et al. (2018) and Bhandari et al. (2021). This is mainly due to the fact that it is even simpler than the Calvo model by assuming a representative firm, but the recent literature shows that there are other theoretical and empirical aspects in favor of the Rotemberg model (see Section 2.2).

This motivates our research question of whether it is possible to capture important aspects of complex state-dependent price-setting models in a simple way by adopting a generalized and thus more flexible cost function of price adjustment. Such a generalization of the Rotemberg model seems particularly attractive given the large cost shocks observed recently and the rapid advances in nonlinear macroeconomic methods and computational power. We will present this generalized cost function in the Section 3.

2.2 Rotemberg versus Calvo: theory and evidence

Our modeling approach builds on the Rotemberg model, because it can be easily generalized to a nonlinear setting. It is necessary to ask whether this model is a reasonable starting point also from an empirical point of view, compared to other simple alternatives, especially the Calvo model. Due to the widespread use of both the Calvo and Rotemberg model in macroeconomics, there is a rich and still active literature comparing and empirically testing both price-setting models. It has been shown that although both models are equivalent up to a first-order approximation around the zero inflation steady state, there can be large differences between the two when non-linearities are taken into account.²

A main difference between the two models is that the Calvo model introduces an additional state variable in the form of price dispersion, for which there is no equivalent in the Rotemberg model. The price dispersion induces misallocation and is the main driver of the welfare costs of inflation in this model. Empirically, it is obvious that there is price dispersion in reality, as not all firms set their prices symmetrically. However, empirical results from Nakamura et al. (2018) suggest that the Calvo model greatly exaggerates price dispersion. They test the implication of the model that if price dispersion increases rapidly with inflation, the absolute size of price changes should also increase with inflation and do not find this confirmed in US data from the 1970s and 1980s. Furthermore, the

²The details of our nonlinear implementation of the Calvo model are given in Appendix A.2.

Calvo model assumes that firms must always satisfy demand, which is inconsistent with profit-maximising behaviour when long-run inflation is positive. Hahn (2022) shows that in a model where firms do not have to satisfy demand, this implies a significantly lower loss in total effective productivity compared to a benchmark model without the possibility of rationing. To summarise, while the Rotemberg model generates no price dispersion, the Calvo model generates too much of it, so that it is not clear which one is closer to reality. In terms of computational complexity of the nonlinear solution, the additional state variable is of course an additional burden.

More generally, the literature presents mixed results regarding the empirical performance of both models. Ascari et al. (2011) come to the conclusion that the Calvo model is empirically superior to the Rotemberg model when the model is estimated in the presence of positive trend inflation. However, as they also report, this only applies if price adjustment costs are interpreted as real resource costs (see also section B.4). Moreover, Ascari (2004) shows that trend inflation in the Calvo model has a large influence on the steady state output and there is a surprisingly low threshold value that trend inflation cannot exceed, depending on the model calibration. This limits the usefulness of the Calvo model when analysing models involving trend inflation. Richter and Throckmorton (2016) find that the Rotemberg model explains the macroeconomic data from 2008-2011 better because it endogenously generates more volatility at the zero lower bound. Similarly, Sims and Wolff (2017) conclude that fiscal multipliers in the Rotemberg model are more volatile between the states of the economy than in the Calvo model. Miao and Ngo (2021), in turn, conclude that both models produce very similar results at the zero lower bound if the price adjustment costs are refunded to households in the Rotemberg model. Furthermore, Oh (2020) shows that the two models have different dynamics in response to uncertainty shocks. In the Rotemberg model, uncertainty shocks lead to a decline in output and inflation, which is consistent with his empirical results. In contrast, uncertainty shocks in the Calvo model lead to a decrease in output but an increase in inflation as firms set higher prices as a precautionary measure. Iania et al. (2023) show that in the Calvo model the inflation cost channel produces the desired term premium moments but has non-trivial, counterintuitive approximation errors in the price dispersion function, whereas the Rotemberg model can successfully accommodate the intuition of the inflation cost channel while maintaining comparable term premium dynamics.

Our conclusion is that there is no consensus in the literature as to which model is empirically preferable. In terms of computational simplicity, the Rotemberg model is much easier to generalize and to use in a nonlinear setting.

3 Generalization of Rotemberg price-setting

In the following, we first describe the profit maximisation problem of firms in general terms, including a brief description of the Rotemberg case as a reference. We then present our proposal of a sigmoid marginal cost function, which is a generalization of the latter as it nests it as a limiting case. Afterwards, we compare the classical and our generalized Rotemberg model to the Calvo model in light of the literature.

3.1 Price-setting by firms facing price adjustment costs

We assume a monopolistically competitive environment where profit-maximising firms face a demand function with constant elasticity $-\varepsilon$. Marginal costs MC_t are independent of output, exogenous to the firm and the same for all firms. Firm i sets its price $P_{i,t}$ and receives $P_{i,t}/(1 + \tau_t)$ for each unit sold, where τ_t is the value added tax (VAT). If the firm changes its price including VAT, it must pay price adjustment costs of $F(P_{i,t}/P_{i,t-1} - 1)Y_tP_t$, where the nominal industry output Y_tP_t serves as adjustment cost base. The firm discounts future profits with the stochastic nominal discount factor $\Lambda_{t,t+j}$. This results in the following dynamic pricing problem:

$$\max_{\{P_{i,t}\}_{t=0}^{\infty}} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left\{ \left(\frac{P_{i,t+j}}{1 + \tau_t} - MC_{t+j} \right) \left(\frac{P_{i,t+j}}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j} - F \left(\frac{P_{i,t+j}}{P_{i,t+j-1}} - 1 \right) Y_{t+j} P_{t+j} \right\}$$

By taking the derivative with respect to $P_{i,t}$ and using that $P_{i,t} = P_t$ in equilibrium because of symmetry, we can write the aggregate first order condition in real terms as

$$\frac{(1 - \varepsilon)}{1 + \tau_t} + \varepsilon mc_t + \Lambda_{t,t+1} f(\pi_{t+1}) (1 + \pi_{t+1})^2 \frac{y_{t+1}}{y_t} = f(\pi_t) (1 + \pi_t), \quad (1)$$

where $mc_t = MC_t/P_t$ denotes real marginal costs inflation is defined as $\pi_t = P_t/P_{t-1} - 1$. In the classical Rotemberg model, the absolute cost function is assumed to be quadratic, $F(\pi) = (\theta_R/2)\pi^2$, which implies that the marginal cost of price adjustment $f(\pi) = \theta_R\pi$ is linear in π .

3.2 The sigmoid marginal cost function

For the marginal cost of price adjustment we propose to use a *sigmoid* function, which is defined as a function on the real line with the following properties:

- it is bounded
- it is monotonically increasing
- it is differentiable

- there is exactly one inflection point x_0 , and $\Sigma(x)$ is convex for $x \leq x_0$ and concave for $x \geq x_0$

We restrict attention to sigmoid functions Σ with inflection point $x_0 = 0$ that are symmetric in the sense that $\Sigma(-x) = -\Sigma(x)$, although one could easily allow for non-symmetric cost functions. We write the marginal cost function in general form as

$$f(\pi) = \theta_B \Sigma\left(\frac{\pi}{\theta_S}\right), \quad \theta_B \geq 0, \quad \theta_S > 0 \quad (2)$$

where $\Sigma : \mathfrak{R} \rightarrow [-1, 1]$ stands for any symmetric sigmoid function normalized such that $\Sigma(0) = 0$, $\Sigma'(0) = 1$ and $\lim_{\pi \rightarrow \infty} \Sigma(\pi) = 1$. The absolute cost function is then given by $F(\pi) = \int_0^\pi f(x)dx$.

The parameters θ_B and θ_S determine the derivative at the origin and the bounds of the marginal cost function. Since $\lim_{\pi \rightarrow \infty} f(\pi) = \theta_B$, we call θ_B the boundary parameter, while we will refer to θ_S as a shape parameter. Marginal costs increase in π with

$$f'(\pi) = \theta_B \frac{d\Sigma\left(\frac{\pi}{\theta_S}\right)}{d\pi} \geq 0 \quad (3)$$

and therefore $f'(0) = \theta_B/\theta_S$. By setting parameters such that

$$\theta_B = \theta_S \theta_R \quad (4)$$

we obtain a marginal cost function with upper bound θ_B and slope θ_R at the origin. This implies that the model is equivalent to the classical linear Rotemberg marginal cost function up to a linear approximation at the origin, so that it delivers the same dynamics as the classical Calvo and Rotemberg models for sufficiently small shocks. To interpret the value of θ_S , notice that it is the inflation level at which the Rotemberg marginal cost equals the upper bound of the generalized Rotemberg marginal cost. Since we calibrate all models in the numerical section to monthly frequency, it is natural to express θ_S in annual terms, $\theta_S^{ann} \equiv (1 + \theta_S)^{12} - 1$.

It is important to realize that symmetry implies $\Sigma''(0) = 0$ and therefore $f''(0) = 0$. In a perturbation around a zero-inflation steady state, the difference between classical and generalized Rotemberg therefore only appears at order 3 or higher. To assess the extent to which perturbation solutions are able to capture the nonlinearities in the model, we will compare perturbation solutions to exact perfect foresight solutions in some of the numerical experiments below.

The marginal cost function (2) nests four pricing models as special or limit cases. (i) If θ_S and θ_B go to infinity such that condition (4) holds, our model converges to the classical Rotemberg model in the sense that $f(\pi) \rightarrow \theta_R$ for any π . (ii) If $\theta_B = 0$, it holds that $F(\pi) = f(\pi) = f'(\pi) = 0$ and thus prices are flexible. (iii) If $\theta_B \rightarrow \infty$ with

$\theta_S = 1$, total costs of price adjustment go to ∞ for any $\pi > 0$, and thus optimal prices are constant in the limit. (iv) If $\theta_S \rightarrow 0$ with $\theta_B > 0$, the marginal cost of price adjustment converges to θ_B for every $\pi > 0$ and to $-\theta_B$ for every $\pi < 0$. The absolute cost function is linear with a kink at $\pi = 0$.

Prominent examples of normalized sigmoid functions are

$$\Sigma_1(x) = \operatorname{erf}\left(\frac{\sqrt{\bar{\pi}}}{2}x\right), \quad \bar{\pi} = \arccos(-1) \quad (5)$$

$$\Sigma_2(x) = \left(\frac{1 - e^{-2x}}{1 + e^{-2x}}\right) = \tanh(x) \quad (6)$$

$$\Sigma_3(x) = \frac{x}{\sqrt{1 + x^2}} \quad (7)$$

$$\Sigma_4(x) = \frac{2}{\bar{\pi}} \arctan\left(\frac{\bar{\pi}}{2}x\right), \quad \bar{\pi} = \arccos(-1) \quad (8)$$

$$\Sigma_5(x) = \frac{x}{1 + |x|} \quad (9)$$

Here, erf denotes the error function, $\Sigma_2(x)$ is a scaled and shifted version of the logistic function, and $\Sigma_3(x)$ is an algebraic function. Since π denotes inflation, as is common in the macroeconomic literature, we write the mathematical constant of the same name in $\Sigma_4(x)$ as $\bar{\pi}$. Functions $\Sigma_1(x)$ to $\Sigma_4(x)$ are all smooth in the sense of infinitely often differentiable, but $\Sigma_5(x)$ is not, it is only once differentiable at the origin. Smoothness is necessary if the solution is to be computed by a perturbation approach.

Any function $\Sigma(x) = 2(F(x) - 0.5)$ with F being the distribution function of a symmetric probability distribution satisfies our definition. Of the examples above, Σ_1 corresponds to the normal distribution, Σ_2 to the logistic distribution and Σ_4 to the Cauchy distribution. Figure 1 graphically illustrates the various sigmoid marginal cost functions and their corresponding absolute cost functions, and compares them to the classic Rotemberg quadratic absolute and linear marginal cost functions. The price changes (in percent and annualized) are plotted on the x-axis and the function value is plotted on the y-axis.

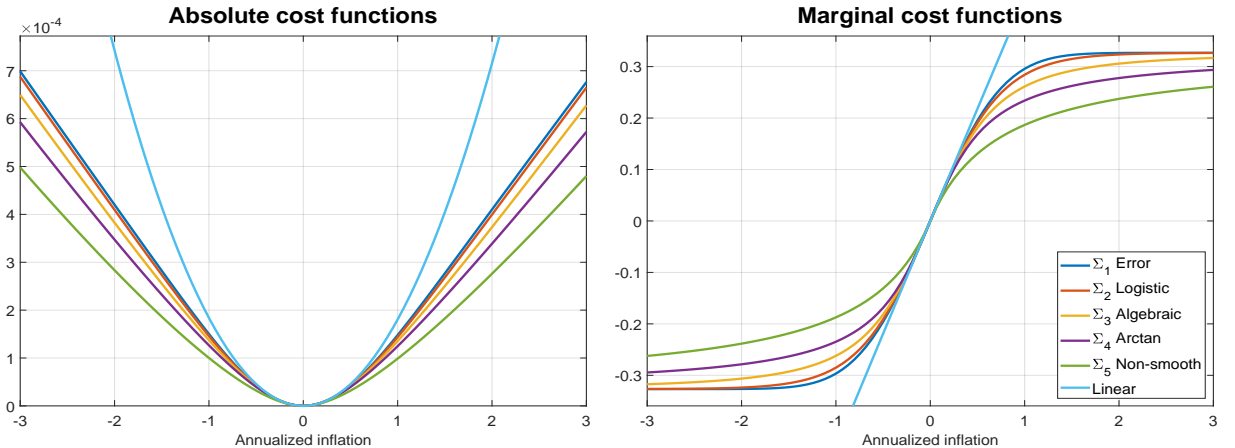


Figure 1: Comparison of various absolute and marginal cost functions

| $\frac{1}{8}\theta_S$ | $\frac{1}{4}\theta_S$ | $\frac{1}{2}\theta_S$ | θ_S | $2\theta_S$ | $4\theta_S$ | $8\theta_S$ | $16\theta_S$ |
|--|-----------------------|-----------------------|------------|-------------|-------------|-------------|--------------|
| Error function Σ_1 | | | | | | | |
| 12.4 | 24.6 | 46.9 | 79.0 | 98.8 | 100.0 | 100.0 | 100.0 |
| Logistic function Σ_2 | | | | | | | |
| 12.4 | 24.5 | 46.2 | 76.2 | 96.4 | 99.9 | 100.0 | 100.0 |
| Algebraic function Σ_3 | | | | | | | |
| 12.4 | 24.3 | 44.7 | 70.7 | 89.4 | 97.0 | 99.2 | 99.8 |
| Arctan function Σ_4 | | | | | | | |
| 12.3 | 23.8 | 42.4 | 63.9 | 80.4 | 90.0 | 94.9 | 97.5 |
| Non-smooth function Σ_5 | | | | | | | |
| 11.1 | 20.0 | 33.3 | 50.0 | 66.7 | 80.0 | 88.9 | 94.1 |

Table 1: Speed of approach to upper bound

Depending on the properties of the corresponding probability distributions, the different sigmoid functions approach the upper bound at different speed. Since the normal distribution has thin tails, $\Sigma_1(x)$ converges quickly to the upper bound. The Cauchy distribution does not have finite moments, so $\Sigma_4(x)$ converges slowly to the upper bound. This is illustrated in Table 1. For example, if a company makes a price change equal to θ_S and its marginal cost function is based on the arc tan function, then it must pay a marginal cost equal to 63.9 percent of the maximum marginal cost θ_B . Despite the differences shown in the table, it turns out that the various sigmoids functions lead to quite similar results after recalibration of the parameters. For this reason, we did not test more examples of sigmoid functions or try to find a continuously parameterized class of functions. In most of the numerical exercises, we will use the arctan function, which approaches the bound relatively slowly, slightly smoothing the response of prices to shock sizes.

| θ_S^{ann} | Annualized inflation rate | | | | | | | |
|------------------|---------------------------|------|------|------|------|------|------|------|
| | 1% | 2% | 3% | 5% | 8% | 10% | 15% | 20% |
| 1% | 0.64 | 0.40 | 0.29 | 0.19 | 0.12 | 0.10 | 0.07 | 0.05 |
| 2% | 0.85 | 0.64 | 0.50 | 0.34 | 0.23 | 0.19 | 0.13 | 0.10 |
| 4% | 0.95 | 0.85 | 0.73 | 0.56 | 0.41 | 0.34 | 0.25 | 0.20 |
| 8% | 0.99 | 0.95 | 0.90 | 0.79 | 0.64 | 0.56 | 0.43 | 0.35 |
| 16% | 1.00 | 0.99 | 0.97 | 0.92 | 0.84 | 0.78 | 0.66 | 0.57 |
| 32% | 1.00 | 1.00 | 0.99 | 0.98 | 0.94 | 0.92 | 0.85 | 0.78 |
| 64% | 1.00 | 1.00 | 1.00 | 0.99 | 0.98 | 0.97 | 0.94 | 0.91 |

Table 2: Marginal cost in Generalized Rotemberg Model, relative to Rotemberg

For the arctan function, Table 2 shows how the marginal price adjustment cost, relative to the marginal cost in the classic Rotemberg model, goes down with the size of the inflation rate, for different values of the annualized shape parameter θ_S^{ann} . For any θ_S , the bound θ_B was chosen such that the implied θ_R corresponds to a Calvo parameter of 0.9, that is 90 percent of firms cannot change their price in a given month.

3.3 Calibrating the parameters of the marginal cost function

The marginal cost function (3) has two free parameters, θ_S and θ_B . While both parameters can be used to match some moments, an alternative calibration strategy exploits the equivalence of the Calvo and Rotemberg models for small shocks around the zero-inflation steady state (see, for example, Keen and Wang, 2007). This can be motivated by the recent finding by Auclert et al. (2024) that in a broad class of menu cost models, the first-order dynamics of aggregate inflation in response to arbitrary shocks to aggregate costs are almost the same as in Calvo models. Setting the Calvo parameter θ_C as one minus the steady state frequency of price adjustment, the corresponding marginal cost of price adjustment is given by $\theta_R = (\varepsilon - 1)\theta_C / [(1 - \theta_C)(1 - \theta_C\beta)]$. This is the typical way of justifying the Rotemberg parameter. Using (4), we obtain θ_B as a function of θ_S or vice versa.

An interesting alternative route follows Alvarez et al. (2016), who show that the cumulative impulse response of the price level to a permanent nominal cost shock in continuous time depends only on the ratio of the kurtosis to the frequency of price changes. Auclert et al. (2024) demonstrate that the resulting formula also provides an excellent approximation in discrete time. Furthermore, they make it clear that the Calvo model must have the same ratio of kurtosis to frequency as the menu cost model in order to have the same cumulative impulse response as the latter. Let the kurtosis and frequency of price changes in the menu cost model be given by kur_{MC} and $freq_{MC}$, respectively, the kurtosis in the Calvo model be given by $kur_C = 3(1 + \theta_C)$ and frequency in the latter is $freq_C = 1 - \theta_C$. Using the just mentioned equality of the ratios of kurtosis and frequency of both models, the following Calvo parameter can be easily derived:

$$\theta_C = \frac{kur_{MC}/freq_{MC} - 3}{kur_{MC}/freq_{MC} + 3} \quad (10)$$

One can obtain frequency and kurtosis of price changes either from a menu cost model that appears as a good approximation to the true data generating process, or calibrate the model directly from the data by substituting the empirical kurtosis and frequency into (10), possibly controlling for unobserved heterogeneity and sales (see e.g. Alvarez et al., 2022; Cavallo et al., 2023).

4 Generalized Rotemberg pricing versus menu costs

Menu cost models have solid micro-foundations, and have become the workhorse for explaining pricing behavior at the micro level. They also imply strongly nonlinear behavior on the aggregate level. Since in general they give rise to firm heterogeneity that cannot be aggregated analytically, they are difficult to integrate into medium- or large

scale macroeconomics models. It would therefore be useful to have a tractable approximation that implies nonlinear aggregate dynamics similar to menu cost models, if properly calibrated. In this section we examine whether GRP satisfies this requirement. The question is not whether GRP explains any stylized facts, but to what extent the generalized Rotemberg pricing model is able to replicate the macroeconomic consequences of menu cost pricing, especially under large shocks and potentially non-zero steady state inflation.

4.1 The menu cost model

To study the effects of different price setting mechanisms in isolation from general equilibrium effects, we consider an industry under monopolistic competition subject to a sudden permanent increase in marginal costs. The industry is populated by a continuum of ex ante identical firms. Firm i is producing output y_i with labor as the only input:

$$y_{i,t} = \xi_{i,t} L_{i,t} \quad (11)$$

The log of firm-specific productivity $\xi_{i,t}$ follows an AR(1) process $\log \xi_{i,t} = \rho \log \xi_{i,t-1} + \varepsilon_{i,t}$ which is independent across firms. The nominal wage w_t is exogenous to the industry, so that the nominal costs of firm i are given by $w_t/\xi_{i,t}$. Monopolistic competition in the industry implies that firm demand is given by $(p_{i,t}/P_t)^{-\epsilon} Y_t$, where the aggregate price index P_t is given by the Dixit-Stiglitz aggregator $P_t = (\int p_{i,t}^{1-\epsilon})^{1/(1-\epsilon)}$. Industry demand Y_t follows $Y_t = P_t^{-\eta}$.

The only endogenous firm-specific state is the nominal price $p_{i,t}$. Changing the price is subject to menu costs $\kappa_{i,t}$ which are potentially stochastic and independent over time and across firms. We treat adjustment costs here as "deliberation costs", affecting the decision of the firm but not costing physical resources or lowering output. Since the model here is in partial equilibrium with exogenous aggregate demand, this does not make a difference for inflation dynamics. Nominal profits are discounted at the constant rate β , implicitly assuming that the dynamics of this specific industry does not affect the stochastic discount factor of the representative share holder.

In the numerical experiments below, we abstract from aggregate uncertainty and assume that the path of industry wages w_t is known. Industry price level P_t and demand Y_t are determined in a perfect foresight equilibrium. Firm i chooses a state-contingent path of prices $p_{i,t}$ and output $y_{i,t}$ to maximize

$$\mathbb{E} \sum_t \beta^t [y_{i,t} (p_{i,t} - w_t/\xi_{i,t}) - \kappa_{i,t} I \{p_{i,t} \Pi^* - p_{i,t-1}\}] \quad (12)$$

where $I \{x\}$ is the indicator function with $I \{0\} = 0$ and $I \{x\} = 1$ for $x \neq 0$. If trend inflation is positive, so $\Pi^* > 1$, we define $p_{i,t}$, P_t and w_t as relative to the trend of the price

level. In a steady state, industry price P_t and industry wage w_t grow at the aggregate rate of inflation Π^* . If firm i does not adjust its price, $p_{i,t}$ diminishes at the rate of inflation, and therefore adjustment costs have to be paid if $p_{i,t}\Pi^* \neq p_{i,t-1}$.

4.2 Industry model with Rotemberg pricing

The industry model with Rotemberg pricing can be summarized in the following three equations:

$$P_t = P_{t-1} \frac{\Pi_t}{\Pi^*} \quad (13)$$

$$Y_t = P_t^{-\eta} \quad (14)$$

$$(1 - \epsilon) + \epsilon \frac{w_t}{P_t} = \theta_B \Sigma(\pi_t/\theta_S) \Pi_t - \beta^{-1} \frac{Y_{t+1}}{Y_t} \theta_B \Sigma(\pi_{t+1}/\theta_S) \Pi_{t+1}, \quad \pi_t \equiv \Pi_t - 1 \quad (15)$$

Eqs.(13) and Equ.(14) express the fact that P_t is defined relative to the trend price level, and this is what industry demand depends on. Equ.(15) is the first order condition for price setting, equivalent to Equ. 20, where $\Sigma(\cdot)$ is one of the sigmoid functions presented in Section 3.2. Notice that marginal adjustment costs are a function of absolute inflation Π , not relative to trend Π^* .

4.3 Functional forms and parameters

For the numerical experiments, we take most parameter values from Costain and Nakov (2019). Autocorrelation of firm productivity is set to $\rho = 0.95$ at monthly frequency, and the variance of the shock process ε is set to $0.06^2(1 - \rho^2)$ so that the unconditional standard deviation of $\log \xi$ equals 0.06. The cost parameter $\bar{\kappa}$ is set in all three cases such that the frequency of price adjustment in the stationary state equals 10.2 percent per month.

For the stochastic price adjustment cost, we consider several distribution functions:

1. **LogN(0.01)**: Log-normal distribution with parameters $\sigma = 0.01$ and mean μ being calibrated to yield 10.2 percent frequency of price adjustment. With σ being very low, this is a slightly smoothed version of a fixed menu cost.
2. **LogN(1.0)**: Log-normal distributed with parameters $\sigma = 1.0$ and mean μ being calibrated to yield 10.2 percent frequency of price adjustment.
3. **LogN(1.0), freq=5.1**: Log-normal distributed with parameters $\sigma = 1.0$ and mean μ being calibrated to yield 5.1 percent frequency of price adjustment.
4. **LogN(3.0)**: Log-normal distributed with parameters $\sigma = 3.0$ and mean μ being calibrated to yield 10.2 percent frequency of price adjustment. The cost distribution

is widely dispersed, a significant fraction of firms faces prohibitive adjustment costs, similar to a Calvo model.

5. **Uniform:** The menu cost is zero with probability $\lambda = 0.051$, and uniformly distributed on the interval $[0, \bar{\mu}]$ with probability $1 - \lambda$, where again μ is calibrated to yield 10.2 percent frequency of price adjustment.

These distributions are illustrated in Fig. 2.

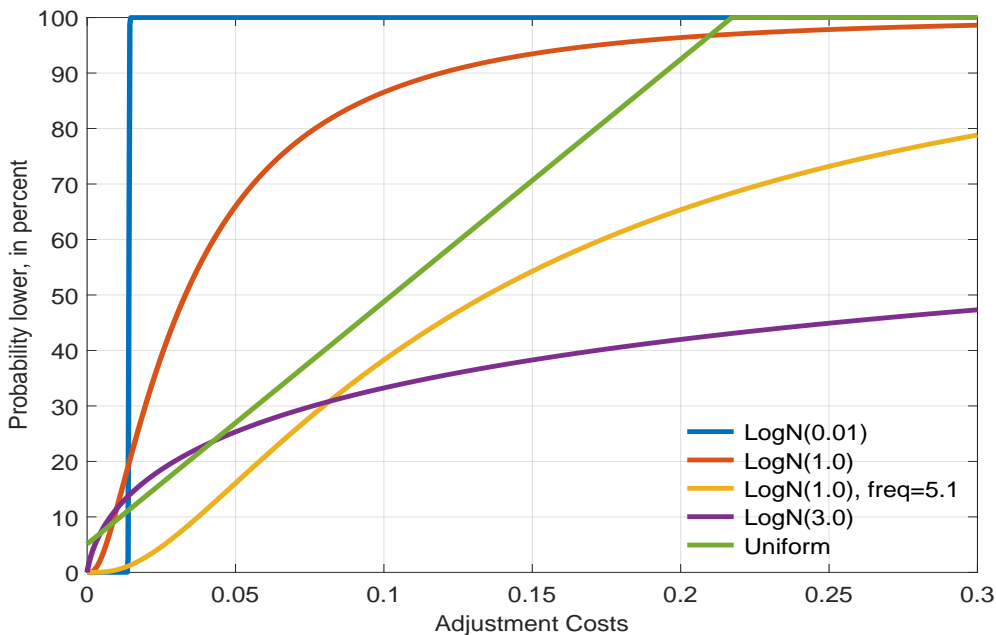


Figure 2: Menu Cost Distributions

4.4 Approximating menu costs with Rotemberg pricing

In the numerical exercise we investigate to what extent the generalized Rotemberg model can mimic the behavior of the menu cost model with respect to the aggregate consequences of a one-time permanent shock to the nominal wage, which is a shock to the nominal marginal cost of each firm in either model. We assume the economy starts out in the stationary state. If there is positive trend inflation, it is understood that nominal price and nominal wage increase with trend inflation in the absence of a shock. Then an unexpected shock hits in period 0 which increases the nominal wage permanently by x percent, where x varies between 1 and 20 percent. We solve for the perfect foresight equilibrium path of P after the shock. In the long run, the industry price also increases by x percent, both in the menu cost and the Rotemberg model. The question is the speed of this adjustment.

For each version of the menu cost model, we calibrate the parameters θ_B and θ_S so as to match the impulse response on impact to the positive 1 percent and 5 percent shock

to marginal costs. Figure 3 shows impulse responses of annualized inflation for different shock sizes ranging from 1 percent to 20 percent. Impulse responses are scaled by the size of the shock, so that they are comparable to the response of a 1 percent shock. In the zero-inflation stationary state, responses to negative shocks are roughly symmetric and therefore not shown here. Rows 1–5 in the graph refer to the five parameterizations of the menu cost distributions as described in Section 4.3. The left column shows the impulse responses of the menu cost model, the middle column those of the calibrated Rotemberg model using the arctan function, the right column those of the Rotemberg model using the erf function. The functions arctan and erf are the two extreme cases among the smooth sigmoid functions shown in Section 3.2.

In all cases, the Rotemberg model was calibrated so as to perfectly match the one-percent and five-percent impulses responses on impact, but the other impulse responses were not targeted. The following main conclusions arise. First, the Rotemberg model captures very well the qualitative feature that larger shocks lead to faster adjustment to the new equilibrium level, in other words, a higher pass-through of the shock. The models differ in how smoothly the pass-through increases with the shock size. In the menu cost model with uniform adjustment costs, and more so with narrowly concentrated lognormal costs, the pass-through goes up to almost 100 percent very quickly for shocks of 10 percent or more. On the opposite side, very widely dispersed lognormal costs show a very smooth increase of the pass-through, similar in this respect to a Calvo model. In general, in Rotemberg models with the error function the pass-through increases more abruptly than with the arctan function, but the different calibrations of the Rotemberg model cannot fully mimic the variations of the menu cost models. Because of the smoothness of the marginal adjustment costs, the impulse responses in the Rotemberg model move relatively smoothly with the shock size. The Rotemberg models come closest to menu cost models with lognormal costs of moderate dispersion (the case $\text{LogN}(1.0)$).

Table 3 displays the parameter values of the different calibrations of the Rotemberg models. While the menu cost models were calibrated to obtain an average frequency of price adjustment of 0.102 percent monthly, which in terms of frequency conforms to a $\theta_R = 0.898$, the calibrated θ_R is considerably lower in all cases. This reflects the fact that a menu cost model implies much higher price flexibility than a Calvo model with the same adjustment frequency, because the endogenous selection of firms who adjust. Even when the menu cost model is calibrated to 0.0051 percent monthly, θ_R is still below 0.898. The case of the wide lognormal distribution is closest to Calvo, which shows in a relatively high value of θ_R . The θ_R 's do not vary much between different sigmoid functions. The shape parameter θ_S^{ann} gives an idea of the curvature of the marginal adjustment cost function. It varies across calibrations between 10 and 20 percent, which means that the classical Rotemberg function would hit the upper bound of the cost function at an annualized inflation rate of 10 to 20 percent. In this range, the cost function becomes

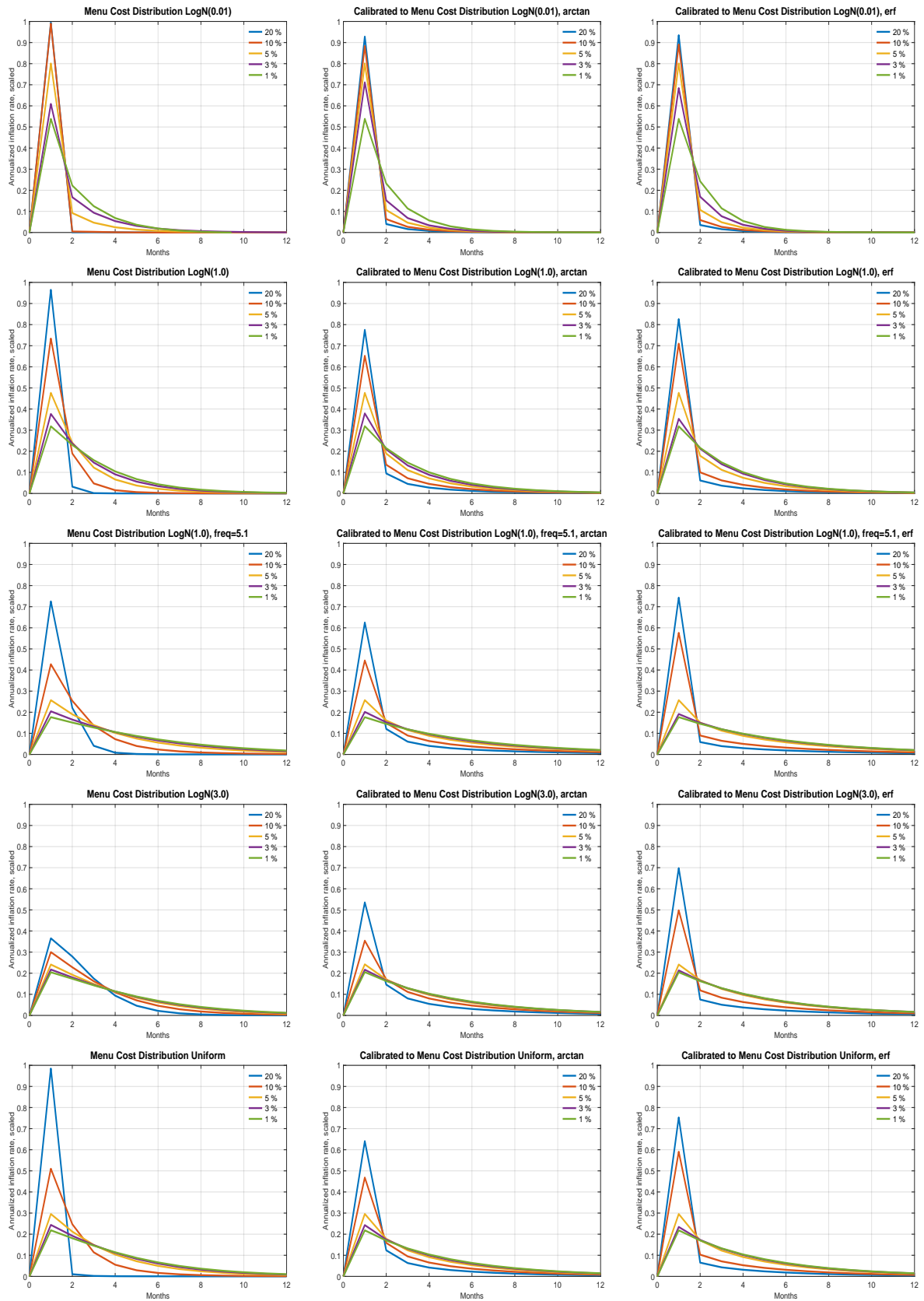


Figure 3: Impulse responses to marginal cost shocks, arctan and erf cost functions

| Model | θ_R , monthly | | | | θ_S^{ann} , in percent | | | |
|--------------------|----------------------|--------|--------|-------|-------------------------------|--------|--------|------|
| | Atan | Algebr | Logist | Erf | Atan | Algebr | Logist | Erf |
| LogN(0.01) | 0.508 | 0.493 | 0.483 | 0.477 | 11.8 | 11.9 | 12.2 | 12.6 |
| LogN(1.0) | 0.690 | 0.688 | 0.687 | 0.686 | 17.2 | 15.2 | 14.2 | 14.1 |
| LogN(1.0),freq=5.1 | 0.828 | 0.827 | 0.828 | 0.827 | 12.0 | 10.4 | 9.4 | 9.2 |
| LogN(3.0) | 0.798 | 0.799 | 0.798 | 0.798 | 21.5 | 17.0 | 14.8 | 13.7 |
| Uniform | 0.786 | 0.785 | 0.785 | 0.785 | 16.3 | 13.8 | 12.3 | 11.8 |

Table 3: Calibrated parameters in Rotemberg model

essentially flat, so that there is almost no inflation smoothing incentive left.

4.5 Variations in trend inflation

In our next experiment, we consider exactly the same models, but now assume a trend inflation rate of about 3 percent annually (the exact number was chosen so as to conform to a integer number of steps in the price grid of the firms). The industry nominal wage follows the same trend. For both models, we use the same calibration as in the zero-inflation steady state, no re-calibration.

Trend inflation introduces an asymmetry: a positive shock of 3 percent now requires a 6 percent change in nominal prices over the course of a year; while total adjustment costs are high, marginal adjustment are relatively flat in this region. Conversely, a negative shock of 3 percent now requires no change in nominal prices after a year, but since the shock is immediate, and inflation accumulates gradually to 3 percent, there is still some adjustment necessary during the year. While total adjustment costs are zero, the cost function is very convex at this point, providing strong incentives to smooth adjustment over time. Comparing the economy with positive trend inflation to the case of zero trend inflation, we can therefore expect that firms react more strongly on impact to a positive shock, less strongly to a negative shock.

Table 4 shows this comparison for positive and negative shocks of 1 and 3 percent, comparing the menu cost model under different distributions of the adjustment costs to the generalized Rotemberg model. Again, the response is scaled by the size of the shock and expressed in percent (i.e., multiplied by 100). For example, the number 7.7 for the Rotemberg model with atan function means that the impact response is higher by 7.7 percent of 1 percent (the shock size), that is 0.006153 compared to 0.00539 in absolute numbers. With negative shocks, a negative number means that the response is less negative under trend inflation. For shock sizes of ± 3 percent, the hypothesis stated above is fulfilled in all cases. Moreover, the Rotemberg model gives quantitatively similar predictions as the Menu cost models. This is remarkable given that both models use the same parameters models as in the zero trend inflation case. For shock sizes of ± 1 percent, changes compared to the zero inflation cases are generally smaller despite the

| Model | ShockSize | Menu | Atan | Algebr | Logist | Erf |
|--------------------|-----------|-------|------|--------|--------|------|
| LogN(0.01) | 1 | -1.4 | 7.7 | 6.0 | 4.4 | 3.5 |
| | -1 | -10.5 | -4.4 | -3.1 | -2.2 | -1.7 |
| | 3 | 9.5 | 5.9 | 6.2 | 6.7 | 7.0 |
| | -3 | -8.2 | -6.9 | -6.7 | -6.7 | -6.9 |
| LogN(1.0) | 1 | 2.3 | 2.9 | 2.3 | 1.8 | 1.4 |
| | -1 | -1.0 | -0.9 | -0.7 | -0.6 | -0.5 |
| | 3 | 4.3 | 5.6 | 5.6 | 5.4 | 5.1 |
| | -3 | -3.7 | -4.9 | -4.4 | -3.8 | -3.3 |
| LogN(1.0),freq=5.1 | 1 | 1.7 | 2.6 | 2.2 | 1.8 | 1.5 |
| | -1 | 0.3 | 0.0 | 0.0 | -0.0 | -0.0 |
| | 3 | 4.0 | 5.2 | 5.3 | 5.3 | 5.1 |
| | -3 | -3.1 | -3.0 | -2.6 | -2.2 | -1.9 |
| LogN(3.0) | 1 | 2.0 | 1.0 | 1.0 | 0.9 | 0.8 |
| | -1 | -1.8 | -0.1 | -0.1 | -0.1 | -0.1 |
| | 3 | 2.1 | 2.2 | 2.3 | 2.2 | 2.2 |
| | -3 | -2.3 | -1.5 | -1.4 | -1.3 | -1.2 |
| Uniform | 1 | 1.5 | 1.9 | 1.7 | 1.4 | 1.2 |
| | -1 | -0.5 | -0.2 | -0.2 | -0.2 | -0.2 |
| | 3 | 3.2 | 4.0 | 4.0 | 3.9 | 3.8 |
| | -3 | -2.6 | -2.7 | -2.5 | -2.2 | -1.9 |

Table 4: Comparison trend inflation 3%

scaling, and the differences between the models are relative larger, with the menu cost model having the opposite sign than what is expected. Since the effects are quantitatively very small, this discrepancy may be due to the discrete approximations of the menu cost. The different sigmoid functions lead to somewhat different impulse responses, but there is no clear pattern with respect to what function leads to results closer to the menu cost model.

To summarize, even for the subtle effects of trend inflation, GRP makes quite similar predictions as the menu cost model.

5 Explaining the Stylized Facts

As described in the introduction, three stylized facts stand out which the Calvo and classical Rotemberg models do not explain. First, the pass-through of large shocks is faster than of small shocks. Second, the higher the trend inflation, the smaller the real effects of nominal shocks. Third, VAT changes are largely passed through at implementation, with little announcement effects.³

³There is another empirical finding that the simple Calvo and Rotemberg models do not explain. Monetary policy shocks have delayed and persistent effects on inflation and output (cf. Friedman and Schwartz (1963), Romer and Romer (1989) and Christiano et al. (1999), among many others). Dotsey

5.1 A basic DSGE model

Following much of the literature, we embed the price setting model into a standard DSGE framework, which is described in more detail in Appendix B [NECESSARY?] **Households** maximize discounted expected lifetime utility with current utility function $C_t^{1-\sigma}/(1-\sigma) - \chi N_t$, where C denotes the consumption bundle and N is household labor supply. Their period budget constraint, expressed in terms of the consumption good, is given by

$$m_t + \frac{b_t}{R_t} = w_t N_t + \frac{m_{t-1} - C_{t-1} + b_{t-1}}{1 + \pi_t} + T_t.$$

where m_t are real money balances of the household, b_t denotes bond holdings, R_t is the nominal interest rate and w_t is the real wage. They receive the lump-sum transfer T_t , which includes the central bank's seignorage profit, the government's VAT revenue $T_t^{VAT} = \tau_t/(1 + \tau_t)Y_t$ and corporate profits. In some model variants with Rotemberg pricing, the household is also reimbursed for the price adjustment costs $F(\pi_t)Y_t$. Monetary policy has real effects because households are subject to the cash-in-advance constraint $C_t = m_t$.

Firms' production function has constant returns to scale, using labor N_t as the only input, with exogenous labor productivity z_t . Real marginal costs are given by $mc_t = \frac{s_t}{z_t}w_t$, where s_t is a price dispersion factor which equals 1 in the Rotemberg model, but is generally larger than 1 in the Calvo model.

Monetary policy sets real money balances according to

$$\frac{m_t}{m_{t-1}} = \frac{\mu e^{u_{m,t}}}{1 + \pi_t}, \quad (16)$$

where μ is the steady state growth rate of money supply, and $u_{m,t}$ is an AR(1) process with autocorrelation ϕ_m . The central bank's seignorage revenue $T_t^m = m_t - \frac{m_{t-1}}{1 + \pi_t}$ is reimbursed to households.

Parameter values. The model period is one month, in line with the menu cost literature. The discount factor β is set to $1.04^{-1/12}$, the steady state growth rate of the money supply equals $\mu = 1.00$, unless stated otherwise. The elasticity of substitution in consumption, ε , is set to 7 in accordance with Costain and Nakov (2019). We follow Nakamura and Steinsson (2010) in setting the intertemporal elasticity of consumption to $\sigma = 1$, and the inverse of the Frisch elasticity of labor supply φ to 0. The latter is common in the menu cost literature. Karadi and Reiff (2019) justify this assumption, for

and King (2005) have shown that such dynamics can occur in a menu cost model with state-dependent pricing, but it does not emerge in simple Calvo or Rotemberg pricing models. The problem applies to GRP as well, as it is equivalent to Calvo and Rotemberg for small inflation rates. However, the literature, starting with Christiano et al. (2005), has taken a different path and has instead shown that these dynamics can also be explained by other model components or rigidities, such as wage rigidities and variable capital utilisation.

example, by the resulting complete long-term pass-through of VAT shocks in line with empirical evidence. The steady state VAT τ is set to 20 percent in section 5.4, otherwise it is set to zero. The parameter χ only scales output and is irrelevant for our purposes. The autocorrelation of monetary shocks is set to $\phi_m = ???$.

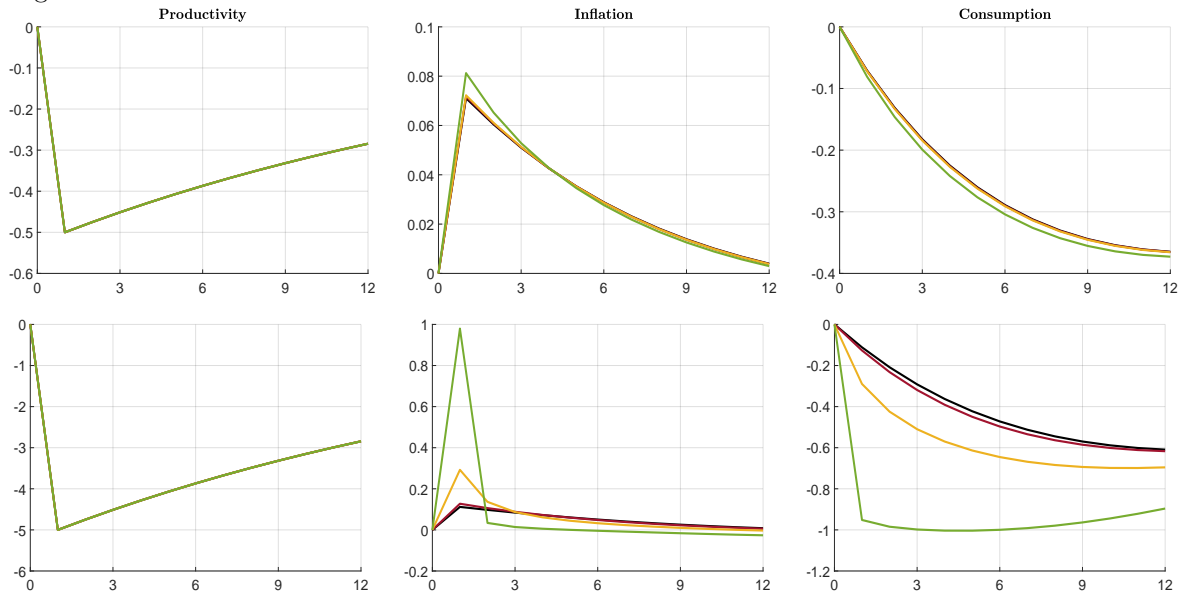
5.2 Large versus small shocks

Figure 4 compares the impulse responses after productivity shocks of the generalized Rotemberg model with different calibrations to those of the Calvo model. The boundary parameter θ_B of the generalized Rotemberg model is always calibrated such that the model is equivalent to the Calvo model to a first order approximation (see section 3.3). The figures then show impulse responses for different shape parameters. When the shape parameter θ_S^{ann} is set to 20 percent (or, in monthly terms, θ_S^{ann} is set to 1.53 percent), the model, at least for the cases considered here, is almost equivalent to the classical Rotemberg and Calvo models. The lower the shape parameter and the larger the shock or the larger the trend inflation, the more the generalized Rotemberg model deviates from the classical Rotemberg and Calvo model.

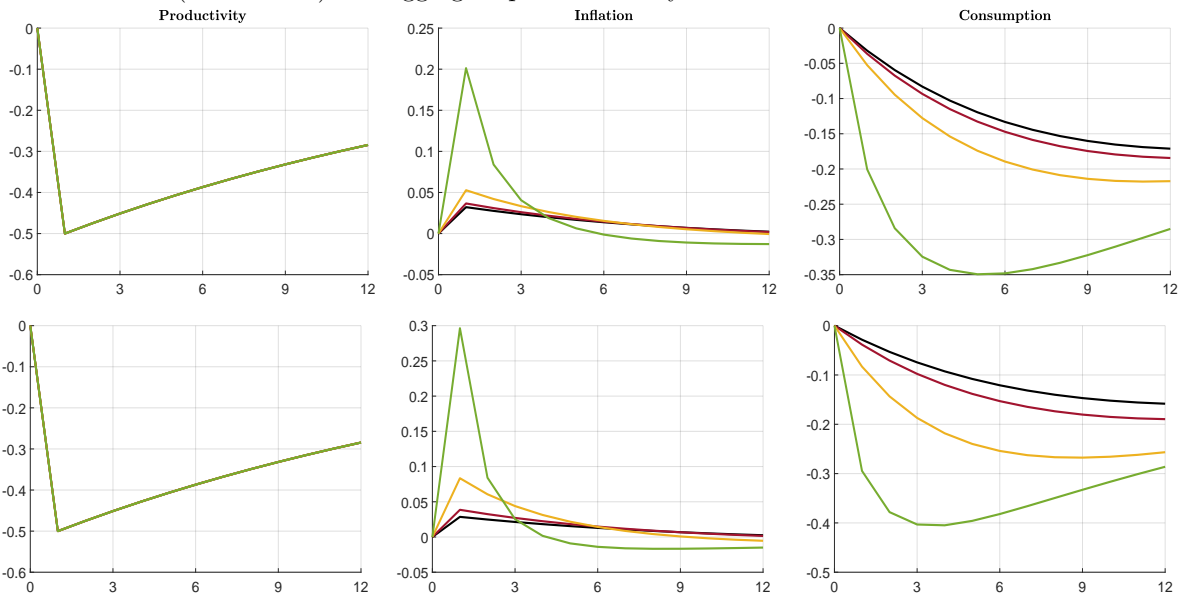
The first two rows of figure 4 compare the impulse responses of the generalized Rotemberg model using different calibrations with those of the Calvo model for large versus small shocks. The impulse responses of inflation and consumption (second and third columns of the figure) are each divided by the size of the shock. The productivity shock has a persistence of 0.95. The first row shows the impulse responses following a small negative productivity shock of 0.5 percent. It can be seen that all model calibrations produce almost the same impulse responses as the Calvo model. The pass-through is about 7 percent in the first period. The Calvo parameter is $\theta_C = 0.8980$, which means that only 10.2 percent of the firms can adjust their prices immediately. However, firms do not fully pass through the shock because, in the Calvo case, they do not know exactly when they will be able to adjust next, and the negative shock slowly phases out. In the Rotemberg model, firms adjust their prices slowly because small price adjustments have low marginal costs. In both cases, therefore, consumers have to reduce their consumption only gradually because of the slow pace of price increases.

The second row of the figure shows the impulse responses to a large, negative productivity shock of 3 percent. Clear differences between the model calibrations can now be observed. While the Calvo model and the generalized Rotemberg model with $(1 + \theta_S)^{12} - 1 = 20\%$ show no difference to the first line, i.e. the pass-through remains at about 7 percent, the variants with shape parameter of 4% or less show considerably larger pass-throughs in the first shock period. Consumers must then also reduce their consumption at a faster rate. The sharp rise in inflation is only short-term. The model variants with a faster pass-through at the beginning show slightly lower inflation rates in

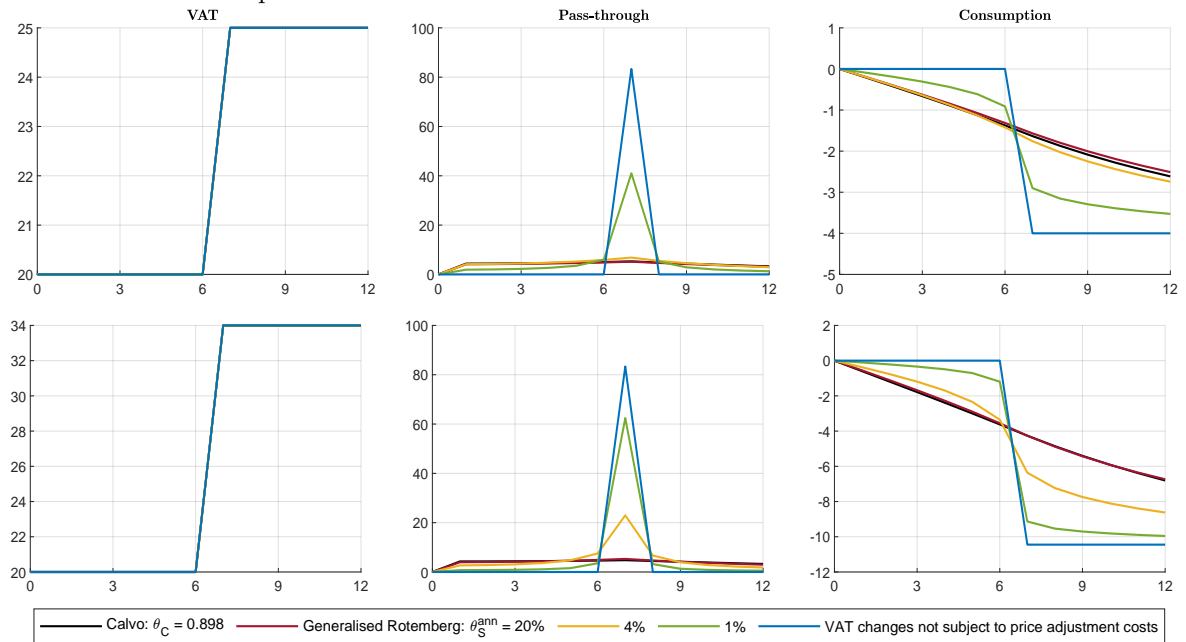
Large versus small shocks



Trend inflation (2% and 4%) and aggregate price flexibility



Announcement and implementation effects of VAT increases



— Calvo: $\theta_c = 0.898$ — Generalised Rotemberg: $\theta_S^{ann} = 20\%$ — 4% — 1% — VAT changes not subject to price adjustment costs

Figure 4: Generalized Rotemberg and the stylized facts of price setting

later months, as prices are already closer to the flexible price optimum.

5.3 Trend inflation and the New Keynesian Phillips curve

There are good reasons to assume that positive trend inflation leads to a steeper New Keynesian Phillips curve (NKPC) or to greater aggregate price flexibility. First, firms have to adjust their prices more often anyway in the presence of trend inflation, and second, trend inflation also provides an argument that firms can use to justify higher price adjustments to their customers. The second aspect in particular seems natural from the point of view of our Rotemberg model, and empirically the frequency of price adjustments in the case of low trend inflation appears to be relatively stable (Klenow and Malin, 2010; Gautier et al., 2024).

By taking the first-order Taylor expansion of equation (20), we can derive the following expression for the NKPC:

$$\hat{\pi}_t = \gamma_{mc} \widehat{mc}_t + \gamma_\pi \hat{\pi}_{t+1} - \gamma_Y \Delta Y_{t+1|t} + \gamma_\tau \hat{\tau}_t \quad (17)$$

The hat represents the percentage deviation of the variable from its steady state or, in the case of inflation, the percentage point deviation from its steady state. $\Delta Y_{t+1|t}$ is in period t expected change in output $(Y_{t+1} - Y_t)/Y$. γ_{mc} , γ_π , γ_Y and γ_τ are parameters that depend on the model and its calibration. Especially relevant for our analysis is γ_{mc} , which determines the slope of the New Keynesian Phillips curve with respect to marginal costs. It is given by

$$\gamma_{mc} = \frac{(\varepsilon - 1)/(1 + \tau)}{f'(\pi)(1 + \pi) + f(\pi)} + \frac{(1 - \beta)f(\pi)(1 + \pi)}{f'(\pi)(1 + \pi) + f(\pi)} \quad (18)$$

With zero trend inflation, this expression becomes $\gamma_{mc} = (\varepsilon - 1)/(f'(0)(1 + \tau)) = (\varepsilon - 1)/(\theta_R(1 + \tau))$, where $\theta_R = \theta_B/\theta_S$. If the slope $f'(\pi)$ of the marginal costs decrease fast enough, as is the case in GRP models with suitable parameters, γ_{mc} is decreasing in inflation, i.e., the Phillips curve is steeper under positive trend inflation. Numerical calculations show that γ_{mc} rises slowly with positive trend inflation until a limit is reached, where it remains constant. Through its influence on the slope of the New Keynesian Phillips curve, trend inflation influences the dynamics of the model already in a first-order approximation, so that the equivalence of GRP and Calvo breaks down even for small shocks.

The third and fourth rows of Figure 4 show impulse responses for small negative productivity shocks of -0.5 %, for levels of trend inflation of 2 % or 4 %. We restrict ourselves here to the case of low trend inflation, as they are typical for developed countries.⁴ We

⁴Low trend inflation allows us to compare the results with the Calvo model. Ascari (2004) shows that in the Calvo model, only low trend inflation rates are compatible with a steady state. And even with trend inflation sufficiently low for a steady state, complex roots and the resulting oscillating behavior

will consider high trend inflation rates in Section 6.

The third row of figure 4 shows impulse responses for 2 percent trend inflation, the fourth row shows impulse responses for 4 percent trend inflation, and the first row of the figure shows the same shock assuming zero trend inflation. It turns out that trend inflation has little effect on the impulse responses of the Calvo model. In GRP model, it depends on the shape parameter: the lower it is, the greater the impact of trend inflation on the impulse responses. Thus, with a low shape parameter, even low trend inflation can significantly steepen the NKPK and thus increase aggregate price flexibility. The higher the trend inflation, the greater the effect on aggregate price flexibility in the generalized Rotemberg model, but the differences between zero and 2 % are larger than the differences between 2 % and 4 % trend inflation. This is in line with discussion of the slope of the NCPK above.⁵

5.4 VAT pass-through

Several studies have recently been published on the pass-through of VAT changes to prices. The authors have come across some clear contradictions to current economic theory. For example, Benzarti et al. (2020) document an asymmetric pass-through, i.e. negative VAT changes are only passed through half as much as positive changes, which can hardly be explained by trend inflation alone.⁶ In addition, they show a very persistent effect of VAT changes that cannot be explained by adjustment costs. They also show large sectoral and firm-specific differences, with their estimated pass-throughs being relatively low between 29 and 55 percent for VAT increases, and only half as high for decreases. In contrast, Karadi and Reiff (2019) report pass-throughs of 33 percent for VAT reductions and 74 to 99 percent for VAT increases. They explain this with a menu-cost model that features trend inflation and fat-tailed product-level shocks. However, this model predicts high price flexibility with respect to monetary policy shocks, so they report doubts about the ability of menu cost models to explain the price rigidities supported by time-series

can occur.

⁵The first result, i.e. the lack of an effect of trend inflation on the impulse responses of the Calvo model, seems to contradict the results of the literature. Ascari and Rossi (2012) find that in the Calvo model the responses of output and inflation become stronger and more persistent when trend inflation increases, even for a first-order Taylor approximation of the model. Moreover, they report a strong effect on output directly in the first period, which then slowly fades out in the following periods. In contrast, we report a hump-shaped impulse response for consumption, which is equal to output in our model. The difference is that, in contrast to Ascari and Rossi (2012), we consider a money supply growth rule. Using their Taylor rule, we arrive at similar results as they do; we show the corresponding impulse responses in the appendix. At this point it should only be emphasized that our main result reported here, that in the generalized Rotemberg model with trend inflation price flexibility increases with lower shape parameters, remains the same. By selecting the shape parameter, one can then obtain a similar or even larger effect of the technology shock than in the Calvo model. However, relative to the effect on impact, the Calvo model always generates a greater persistence of this effect.

⁶It has been known since Ball and Mankiw (1994) that positive trend inflation can explain an asymmetric pass-through, at least to a certain extent and in the short term.

evidence. Benedek et al. (2020) document large differences in the pass-through of VAT shocks, estimating the total pass-through to be only about 25 percent when all VAT changes are treated equally. On closer analysis, they come to the conclusion for the standard rate VAT that even full pass-through cannot be ruled out, while for the reduced rates, for example, there is only a pass-through of about 30 percent.

Our aim here is not to resolve the major discrepancies in empirical estimates or to resolve the inconsistencies with existing models, but rather to show that generalized Rotemberg pricing provides a flexible, yet simple and unified framework to study macroeconomic consequences of VAT changes. Notice that one can generate a very high pass-through directly in the implementation period by assuming that price adjustments in the Calvo or Rotemberg models do not refer to after tax prices, but to before tax prices, so that passing on VAT changes does not imply any costs. Alternatively, the modeler can assume that firms have to bear the full adjustment costs for VAT changes, as for all other price changes. As Karadi and Reiff (2019) document, and is confirmed in the results for the Calvo model below, this implies a low pass-through in the implementation period and comparatively large anticipatory and follow-up effects. The literature cited above largely agrees that there is little evidence of price adjustments in the run-up to a VAT change, i.e. after announcement but before implementation, that a large part of the pass-through occurs immediately upon implementation, and that almost all price changes are completed after a few months.

Figure 4 shows simulation results for 5 percent VAT increases (fifth row) and 14 percent VAT increases (sixth row) announced 6 months before implementation. The order of magnitude corresponds to that examined in the literature, e.g. 14 percent VAT changes in the Finnish hairdressing industry were examined by Benzarti et al. (2020), 5 percent changes were examined by Karadi and Reiff (2019). As can be seen in the second column, the pass-through, defined as $(\pi_t - \pi)/|\Delta\text{VAT}|$, is always below 100 percent during the implementation period. This is because the model predicts that part of the incidence of the tax will be borne by producers, they will reduce their producer prices somewhat and therefore the total pass-through remains below 100 percent. The column also shows that the generalized Rotemberg model implies a pass-through immediately after implementation that, depending on the shape parameter, lies between the low pass-through implied by the Calvo model and the high pass-through implied by a model in which VAT changes do not cause price adjustment costs. This has significant effects on consumption, as the third column shows. While consumption in the Calvo model evolves very smoothly over time, so that the implementation period is not recognizable as such, a model in which VAT changes do not cause price adjustment costs leads to a sudden change when the VAT change is implemented. Consumption in the generalized Rotemberg model falls between the two extremes.

5.5 Solving Generalized Rotemberg models by perturbation methods

The impulse responses presented in this section so far have been obtained using Dynare's perfect foresight solver. For stochastic simulations, the most widely used solution method is perturbation around the steady state. Since the generalized Rotemberg pricing implies strong nonlinearities, we want to investigate how well those are captured by perturbation solutions. In Figure 5 we compare the impulse responses after a negative productivity shock; the top row shows a large negative productivity shock of -3 % and zero trend inflation, while the bottom row shows a small productivity shock of -0.5 % with trend inflation of 4 %. In order to make the impulse responses obtained by perturbation comparable to the perfect foresight case, we compute the perturbation under the assumption of zero shock variance, to eliminate precautionary behavior of firms. Under these assumptions, the perturbation solution is an approximation of the perfect foresight solution.

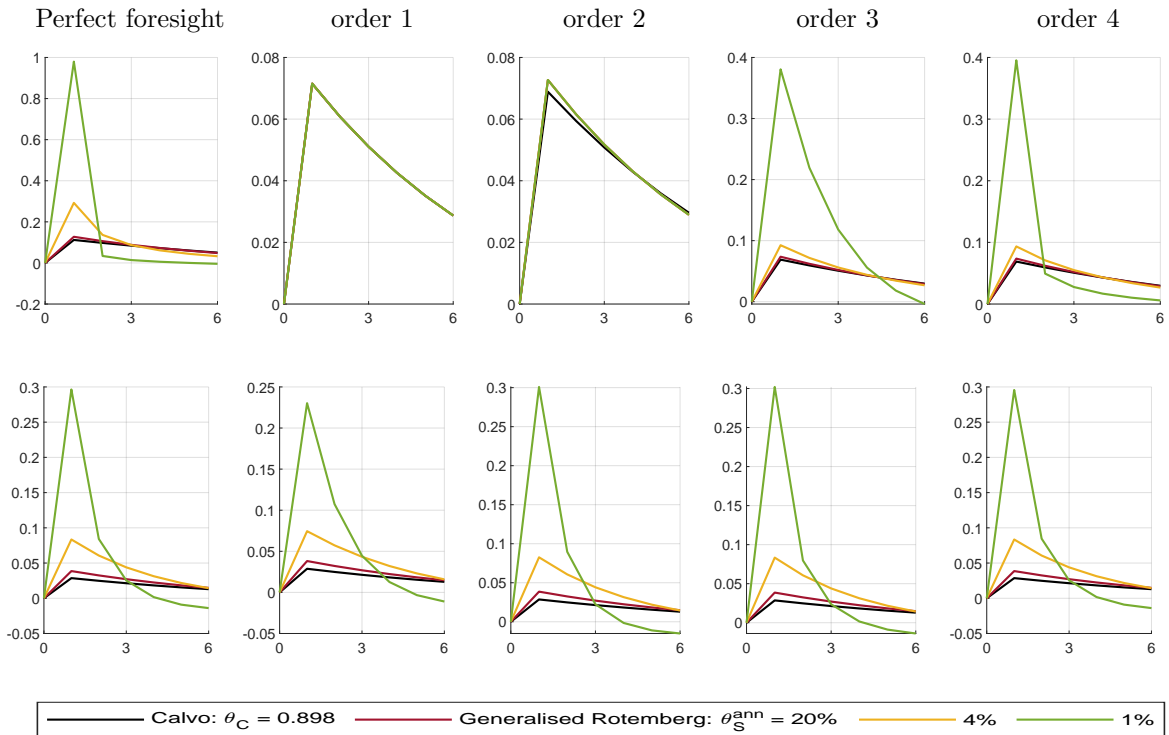


Figure 5: Perfect foresight versus perturbation solutions

The first column of Figure 5 shows the previously presented impulse response using the perfect foresight solution. In both cases, the lower θ_S , the more abrupt the inflation response. The second column shows the results obtained by first order perturbations. In the case of the large shock, the results for the different model variants do not differ; the curves lie exactly on top of each other. The Rotemberg and Calvo models are exactly equivalent here, and the different calibrations of the Rotemberg model are lost due to the

approximation. On the other hand, non-zero trend inflation leads to different results even when a first order perturbation is used. The reason for this has already been explained in Section 5.3 trend inflation affects the parameter of the New Keynesian Phillips curve.

For the large shock, the second-order perturbation leads to almost the same result as the first-order perturbation, although there are slight differences between the Rotemberg and Calvo models. Only from the third-order perturbation onwards are there clear differences, with the response of inflation and consumption being smoothed over time to a much greater extent than in the perfect foresight solution. The fourth-order perturbation provides impulse responses that are close in shape to the perfect foresight solution, but are quantitatively much too small. In contrast, the impulse responses after a small shock and 4 % trend inflation are quantitatively almost indistinguishable from the perfect foresight results already using a second order perturbation.

6 High Levels of Trend Inflation

In this section, we use Rotemberg pricing to reproduce a result from Costain and Nakov (2019), who show how price flexibility increases with higher trend inflation. This case is interesting because it considers very high trend inflation of up to 80 percent, which is the range actually observed in Mexico. Costain and Nakov offer a new theoretical perspective by modeling price rigidity as the result of costly, error-prone decision making, rather than of price adjustment costs. Apart from price setting, their model framework is very similar to ours, so that a comparison requires only a few adjustments. In the following we want to investigate whether our generalized Rotemberg model is able to deliver similar results at the aggregate level to Costain and Nakov’s comparatively complex model for both low and very high rates of trend inflation.

To adapt our model framework to Costain and Nakov (2019), we replace the cash-in-advance constraint and determine liquidity demand by including money in the utility function. Households then maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi N_t + \nu \ln \left(\frac{M_t}{P_t} \right) \right],$$

subject to

$$C_t + m_t + \frac{b_t}{R_t} = w_t N_t + \frac{m_{t-1} + b_{t-1}}{1 + \pi_t} + T_t.$$

The only difference to our previous model is that we replace the cash-in-advance constraint with the following first-order condition for money demand:

$$\frac{\nu}{m_t} = 1 - \frac{1}{R_t} \tag{19}$$

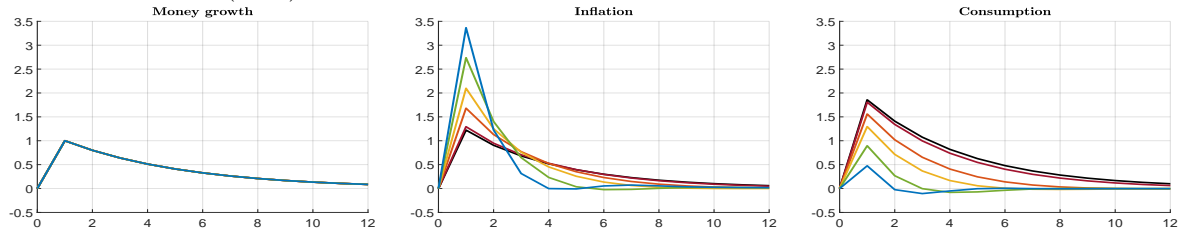
We set $\nu = 1$, $\sigma = 2$ and $\chi = 6$ as in Costain and Nakov (2019).

Figure 6 displays impulse responses of inflation and consumption after a 1 percentage point money supply growth shock with a persistence of $\rho_{u_m} = 0.8$, for different trend inflation rates of 0 to 80 per cent. The first row shows the results of Costain and Nakov (2019), exactly replicated with their model. It can be seen that the higher the trend inflation, the stronger the response of inflation and the weaker the response of consumption.

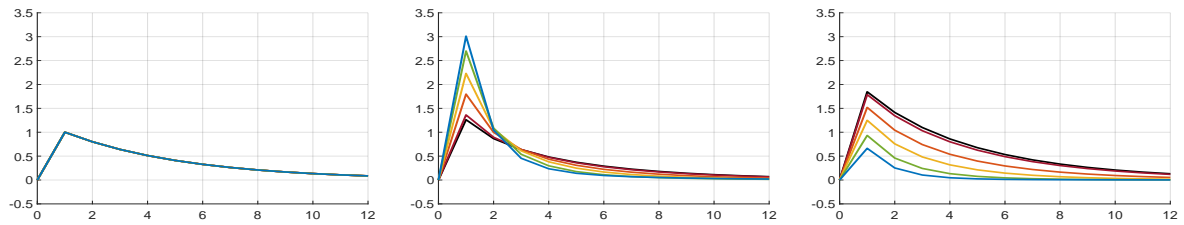
For the results in the second line of the figure, we set our parameters θ_B and θ_S such that the mean square deviation between our inflation impulse responses and those of Costain and Nakov (2019) in the first 12 months after the shock is minimised. This results in $\theta_B = 1.85$ and $\theta_S = 0.0085$, using the generalized Rotemberg model with arctan function. The model matches the impact response very well, just over 1 percentage point increase in inflation at zero or 2 percent trend inflation and over 3 percentage points at 80 percent trend inflation. However, the Costain and Nakov (2019) model generates less persistence at high inflation, with inflation even becoming slightly negative after 5 months. The generalized Rotemberg model, on the other hand, shows a very smooth impulse response. The two models are also close to each other for consumption, although the differences between consumption at high versus low trend inflation are somewhat smaller in the generalized Rotemberg Model.

The success of GRP becomes clearer if we compare it to other models. The third row shows the results of a classic Rotemberg model calibrated according to the Calvo parameter. Inflation reacts only very weakly, while consumption reacts relatively strongly and trend inflation has hardly any influence on the results. The fourth row shows the same classic Rotemberg model, but the parameter θ_R is calibrated using impulse response matching, just as described above for the generalized Rotemberg model. The result is that θ_R is set very small, which results in high price flexibility and the impulse response runs approximately in the middle of the impulse responses of Costain and Nakov (2019). However, it remains the case that trend inflation has no influence on the impulse responses. The last two lines then show the same exercise as in lines 2 and 3, only this time under the assumption that price adjustment costs are real resource costs and thus appear in the market clearing equation. This implies negative corporate profits for high trend inflation. The impulse responses of the classic Rotemberg model now show the opposite relationship between price flexibility and trend inflation. Higher trend inflation accordingly leads to lower price flexibility. The generalized Rotemberg model, on the other hand, continues to produce very similar results to Costain and Nakov (2019); there is only a slight deterioration in the mean square deviations compared to the model with reimbursement of price adjustment costs.

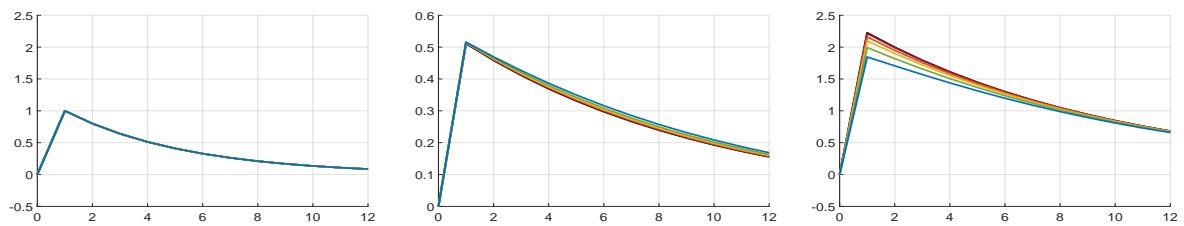
Costain and Nakov (2019)



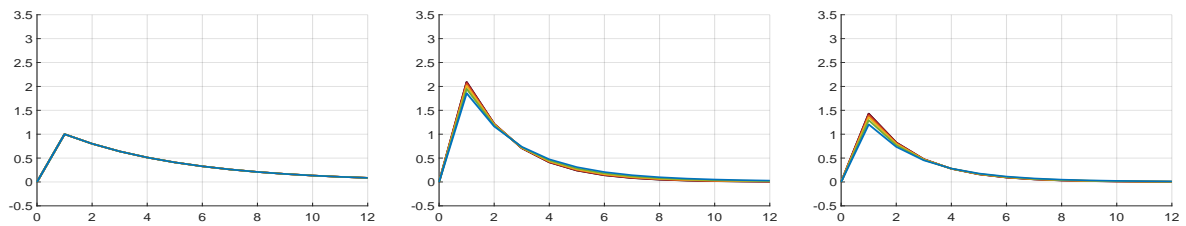
Generalized Rotemberg



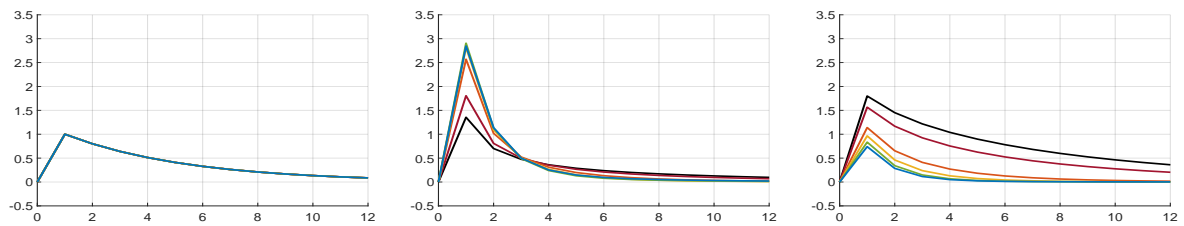
Classical Rotemberg: Calibration according to Calvo parameter



Classical Rotemberg: Optimized calibration



Generalized Rotemberg: No reimbursement of price adjustment costs



Classical Rotemberg: No reimbursement

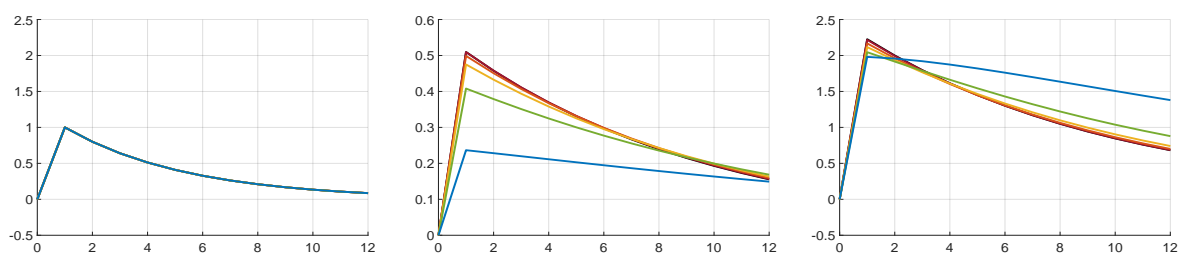


Figure 6: IR to 1 percent money supply shocking, different levels of trend inflation

7 Conclusion

We have proposed a generalized Rotemberg pricing scheme and have shown that it can account for important aspects of the observed nonlinear behavior of price adjustment at the macroeconomic level, such as a higher pass-through in response to larger shocks, a positive impact of trend inflation on price flexibility, and the relationship between announcement and implementation effects. Furthermore, we have shown that with an appropriate calibration of the model, it can generate similar effects on macroeconomic variables as standard versions of the menu cost model. It also generates effects of trend inflation on price flexibility very similar to a recent, substantially more complex model of logit-price dynamics.

There are obvious limitations to the generalized Rotemberg model. If a model of heterogeneous firms with idiosyncratic shocks is the "true" model, our representative firm model is not structural in the sense of Lucas (1976). This shows, for example, in an exaggerated asymmetry between positive and negative shocks in the case of positive trend inflation. To what extent this problem may be resolved by allowing for asymmetric sigmoid functions may be an interesting question for future research.

We see a major application of our price-setting scheme in multi-industry DSGE models, because shocks at the industry level are larger than aggregate shocks, so that nonlinearities become very important. The size of these models makes it very difficult to handle heterogeneous firms, so that the tractability of the generalized Rotemberg model is a big advantage. Estimating such a model is part of our research agenda.

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Appendix: Generalized Rotemberg Price-Setting

A Alternative price-setting models or model variants

A.1 Price-setting by firms when price adjustments due to VAT changes are exempt from price adjustment costs

In section 5.4 we show results for a Rotemberg model in which price adjustments due to VAT changes are exempt from price adjustment costs. The firm's problem is similar to that described in section 3.1. The only difference is that firm i pays the price adjustment costs on changes in the net price instead of the gross price. It must therefore pay price adjustment costs of $F((1 + \tau_t)P_{i,t}/(1 + \tau_{t-1})P_{i,t-1} - 1)Y_tP_t$. The problem becomes:

$$\max_{\{P_{i,t}\}_{t=0}^{\infty}} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left\{ \left(\frac{P_{i,t+j}}{1 + \tau_{t+j}} - MC_{t+j} \right) \left(\frac{P_{i,t+j}}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j} - F \left(\frac{(1 + \tau_{t+j-1})P_{i,t+j}}{(1 + \tau_{t+j})P_{i,t+j-1}} - 1 \right) Y_{t+j} P_{t+j} \right\}$$

By taking the derivative with respect to $P_{i,t}$ and using that $P_{i,t} = P_t$ in equilibrium because of symmetry, we can write the aggregate first order condition in real terms as

$$\frac{(1 - \varepsilon)}{1 + \tau_t} + \varepsilon mc_t + \Lambda_{t,t+1} f(\tilde{\pi}_{t+1}) (1 + \tilde{\pi}_{t+1}) (1 + \pi_{t+1}) \frac{y_{t+1}}{y_t} = f(\tilde{\pi}_t) (1 + \tilde{\pi}_t), \quad (20)$$

where net inflation is defined as $\tilde{\pi}_t = (1 + \tau_t)P_{i,t}/(1 + \tau_{t-1})P_{i,t-1} - 1$.

A.2 The Nonlinear Calvo Model

If firms are subject to Calvo pricing, only a fraction $(1 - \theta_C)$ of these firms can change their price each period. Under the same conditions as above, but without price adjustment costs, the problem of optimal price setting of a firm i that is able to set its price in a given period t is given by

$$\max_{\{P_{i,t}\}_{t=0}^{\infty}} E_t \sum_{j=0}^{\infty} (\theta_C)^j \Lambda_{t,t+j} \left\{ \left(\frac{P_{i,t+j}}{1 + \tau_t} - MC_{t+j} \right) \left(\frac{P_{i,t+j}}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j} \right\}$$

the first order condition of the problem can be written recursively as

$$\varepsilon g_{1,t} = (\varepsilon - 1) g_{2,t}, \quad (21)$$

with

$$g_{1,t} = mc_t Y_t + \theta_C E_t \left\{ \Lambda_{t,t+1} (1 + \pi_{t+1})^{\varepsilon-1} g_{1,t+1} \right\} \quad (22)$$

and

$$g_{2,t} = \frac{p_t^*}{(1 + \tau_t)} Y_t + \theta_C E_t \left\{ (1 + \pi_{t+1})^\varepsilon \Lambda_{t,t+1} \frac{p_t^*}{p_{t+1}^*} g_{2,t+1} \right\} \quad (23)$$

$p_t^* = P_t^*/P_t$ is the price set by the firms that can change their price in period t . It evolves according to

$$p_t^* = \left[\frac{1}{1 - \theta_C} - \frac{\theta_C}{1 - \theta_C} \left(\frac{1}{1 + \pi_t} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \quad (24)$$

In the case of flexible prices, the respective first-order conditions, i.e. either in the Rotemberg price setting equation (20) or in the Calvo price setting equation (26), are reduced to the usual price rule, in which all firms set the same price as a mark-up on their marginal costs, given by $p_t^* = \frac{\varepsilon}{\varepsilon-1} (1 + \tau_t) mc_t = 1$. The Calvo model additionally implies a price dispersion term in the aggregate production function, resulting from the different labour demands of firms. Specifically, if the production function of each firm i is given by $Y_{i,t} = z_t N_{i,t}$, where z_t is aggregate productivity, then the aggregate demand for labour is $N_t = \int_0^1 N_{i,t} d i = \int_0^1 \left(\frac{Y_{i,t}}{z_t} \right) d i = \frac{Y_t}{z_t} \int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^\varepsilon d i$. We can define the price dispersion term as $s_t = \int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^\varepsilon d i$, which becomes a state variable in the model with dynamic equation

$$s_t = \theta_C (1 + \pi_t)^\varepsilon s_{t-1} + (1 - \theta_C) (p_{i,t}^*)^{-\varepsilon}. \quad (25)$$

In the Rotemberg model, all firms can set their optimal price, i.e. the fraction of firms that are not allowed to change their price is $\theta_C = 0$. Thus, in this case, $p_{i,t}^*$ and s_t must always be one, as can be seen in equations (29) and (30).

A.3 The Calvo model, when VAT changes are automatically applied to all prices

As mentioned above, in section 5.4 we show results for a Rotemberg model in which price adjustments due to VAT changes are exempt from price adjustment costs. In this case, the pass-through occurs entirely when the VAT change is implemented. The same can be achieved by a Calvo model in which VAT changes lead directly to price changes, i.e. all prices adjust automatically when the tax changes. In this case, we can define the net price as $\tilde{P}_{t,i} = P_{t,i}/(1 + \tau_t)$. Then we can express the problem of the firm that adjusts its price in period t as follows:

$$\max_{\{P_{i,t}\}_{t=0}^{\infty}} E_t \sum_{j=0}^{\infty} (\theta_C)^j \Lambda_{t,t+j} \left\{ \left(\tilde{P}_{t,i} - MC_{t+j} \right) \left(\frac{(1 + \tau_{t+j}) \tilde{P}_{t,i}}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j} \right\}$$

the first order condition of the problem can be written recursively as

$$\varepsilon g_{1,t} = (\varepsilon - 1) g_{2,t}, \quad (26)$$

with

$$g_{1,t} = \frac{mC_t}{(1 + \tau_t)^\varepsilon} Y_t + \theta_C E_t \{ \Lambda_{t,t+1} (1 + \pi_{t+1})^{\varepsilon-1} g_{1,t+1} \} \quad (27)$$

and

$$g_{2,t} = \frac{p_t^*}{(1 + \tau_t)^{1+\varepsilon}} Y_t + \theta_C E_t \left\{ (1 + \pi_{t+1})^\varepsilon \Lambda_{t,t+1} \frac{p_t^*}{p_{t+1}^*} \frac{(1 + \tau_{t+1})}{(1 + \tau_t)} g_{2,t+1} \right\} \quad (28)$$

$p_t^* = P_t^*/P_t$ is the price set by the firms that can change their price in period t . It evolves according to

$$p_t^* = \left[\frac{1}{1 - \theta_C} - \frac{\theta_C}{1 - \theta_C} \left(\frac{(1 + \tau_t)}{(1 + \tau_{t-1})} \frac{1}{(1 + \pi_t)} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \quad (29)$$

Price dispersion becomes

$$s_t = \theta_C \left[(1 + \pi_t) \frac{(1 + \tau_{t-1})}{(1 + \tau_t)} \right]^\varepsilon s_{t-1} + (1 - \theta_C) (p_t^*)^{-\varepsilon}. \quad (30)$$

B A basic DSGE model

We embed the pricing mechanisms described in section 3 in a general equilibrium framework that is kept as simple as possible. Our aim is to compare generalized Rotemberg pricing with other pricing mechanisms, and these, especially the complex, micro-data motivated pricing models, are usually also integrated into otherwise simple general equilibrium models.

B.1 Households

Households maximize their utility by choosing consumption and leisure subject to a budget and a cash-in-advance constraint. The representative household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\varphi}}{1+\varphi} \right],$$

where C_t is the consumption bundle, N_t is labor supplied by the household, σ and φ are the corresponding elasticities of substitution with respect to consumption and labor, χ is an index that we use to calibrate steady state labor supply and β denotes the discount factor. The household can choose between an infinite number of consumer products, and substitutes them with the elasticity ε . This is the well-known Dixit-Stiglitz framework and ultimately implies the firms' markup price setting. The household's real period

budget constraint is given by

$$m_t + \frac{b_t}{R_t} = w_t N_t + \frac{m_{t-1} - C_{t-1} + b_{t-1}}{1 + \pi_t} + T_t.$$

m_t are real money balances of the household, b_t denotes bond holdings, R_t is the nominal interest rate and w_t is the real wage. T_t denotes the lump-sum transfers to the household, which include the central bank's seignorage profit T_t^m , the government's VAT revenue $T_t^{VAT} = \tau_t/(1 + \tau_t)Y_t$ and corporate profits. In some model variants with Rotemberg pricing, the household is also reimbursed for the price adjustment costs $F(\pi_t)Y_t$. Moreover, we follow Blanco et al. (2024) and Rotemberg (1987) by assuming that the household is also subject to a cash-in-advance constraint given in real terms by

$$C_t = m_t.$$

In this form, the cash-in-advance constraint has a small effect on the results, in particular it does not distort the choice of labor supply (see Blanco et al., 2024), it is almost always binding, at least as long as the interest rate does not become negative, and it is in principle close to the simple quantity equation and is therefore easy to interpret. This is very convenient for later examining the pass-through of money supply growth shocks into prices.

The household's first order conditions are

$$w_t = \frac{\chi N_t^\varphi}{C_t^{-\sigma}}, \quad (31)$$

$$C_t^{-\sigma} = \beta R_t E_t \left[\frac{C_{t+1}^{-\sigma}}{(1 + \pi_{t+1})} \right] \quad (32)$$

Equation (31) equates the real wage with the marginal rate of substitution between consumption and labor and equation (32) is the standard Euler equation, i.e. the optimality condition for the one-period holding of bonds. The latter also determines the stochastic discount factor $\Lambda_{t,t+1}$, which firms use to discount their future profits.

B.2 Firms

Firms use the labor supplied by households and produce the output Y_t with productivity z_t . As shown in the section 2.2, it is also necessary to take the price dispersion s_t into account in the Calvo model, whereby the latter is always equal to 1 in the Rotemberg model. Thus, the aggregate production function can be written as

$$Y_t = \frac{z_t}{s_t} N_t. \quad (33)$$

Cost minimization yields the following expression for marginal costs:

$$mc_t = \frac{s_t}{z_t} w_t \quad (34)$$

The profit maximization is described in the section 3.1 and leads to equation (20) as a first-order condition in the Rotemberg model and to equation (26) in the Calvo model.

B.3 Monetary policy

Real money balances evolve according to

$$\frac{m_t}{m_{t-1}} = \frac{\mu e^{u_{m,t}}}{1 + \pi_t}, \quad (35)$$

where μ is the steady state growth rate of money supply and $u_{m,t}$ is a shock process to monetary policy which is given by

$$u_{m,t} = \phi_m u_{m,t-1} + \epsilon_t^{u_m} \quad (36)$$

with $0 < \phi_m < 1$ and $\epsilon_t^{u_m} \sim i.i.d.N(0, \sigma_m^2)$. The central bank's seignorage revenue $T_t^m = m_t - \frac{m_{t-1}}{1 + \pi_t}$ is reimbursed to households.

B.4 Market clearing

If price adjustment costs correspond to a loss of real resources, then this implies for the goods market clearing that all goods produced are either consumed or used to pay for price adjustments:

$$Y_t = C_t + F(\pi_t) Y_t \quad (37)$$

The problem is that the costs of price adjustments can occasionally be very high, especially in the case of quadratic price adjustment costs. Eggertsson and Singh (2019) show that the Rotemberg model can therefore not generate the Great Depression, because a 10 percent deflation, as observed in the Great Depression, would absorb over 100 percent of output. If, on the other hand, firms only behave as if there were price adjustment costs, or if these are reimbursed to households, all goods produced are also consumed:

$$Y_t = C_t \quad (38)$$

Already Ascari and Rossi (2012) have suggested such an interpretation of price adjustment costs, as this improves the empirical performance of the Rotemberg model in the presence of trend inflation. In the following, we always use the model variant with market clearing

according to equation (38) unless explicitly stated otherwise. See also the discussion in Eggertsson and Singh (2019) on the consequences for the slope of AS and AD equations and the interpretation of price adjustment costs. In the Calvo case, market clearing is achieved in any case by equation (38). Since bonds are in zero net supply, symmetry of the equilibrium implies $b_t = 0$.

B.5 Calibration

In the following, we first briefly describe the calibration of the parameters of the model framework. Note that we parameterize our model to a monthly frequency, in line with the menu cost literature. We then go into detail on the calibration of the price adjustment cost parameters.

B.5.1 Parameters of the model framework

The discount factor β is set to $1.04^{-1/12}$, the steady state growth rate of the money supply $\mu = 1.00$, unless stated otherwise, and the elasticity of substitution ε is set to 7 in accordance with Costain and Nakov (2019). We follow Nakamura and Steinsson (2010) in setting the intertemporal elasticity of consumption $\sigma = 1$ and the Frisch elasticity of labor supply φ to 0. The latter is common in the menu cost literature. Karadi and Reiff (2019) justify this assumption, for example, by the resulting complete long-term pass-through of VAT shocks in line with empirical evidence. We set χ in a way that steady state labor supply is roughly 1/3. The steady state VAT τ is set to 20 percent in section 5.4, otherwise it is set to zero.

C Monetary policy conducted by means of a Taylor rule

D Computational Details

D.1 Menu cost model of Section 4.1

The problem is to compute an industry equilibrium path from one stationary state to a new stationary state after a permanent cost shock. Given a time path of industry price level P_t (or a fixed level P^* for the stationary state), the firm problem is solved by discrete dynamic programming on a finite grid of 501 points in the endogenous state (price) and 21 points in the exogenous state (firm productivity). To achieve smoothness and guarantee existence of an industry equilibrium, the solution is slightly perturbed by allowing that several actions (prices) are taken with positive probability if their level of

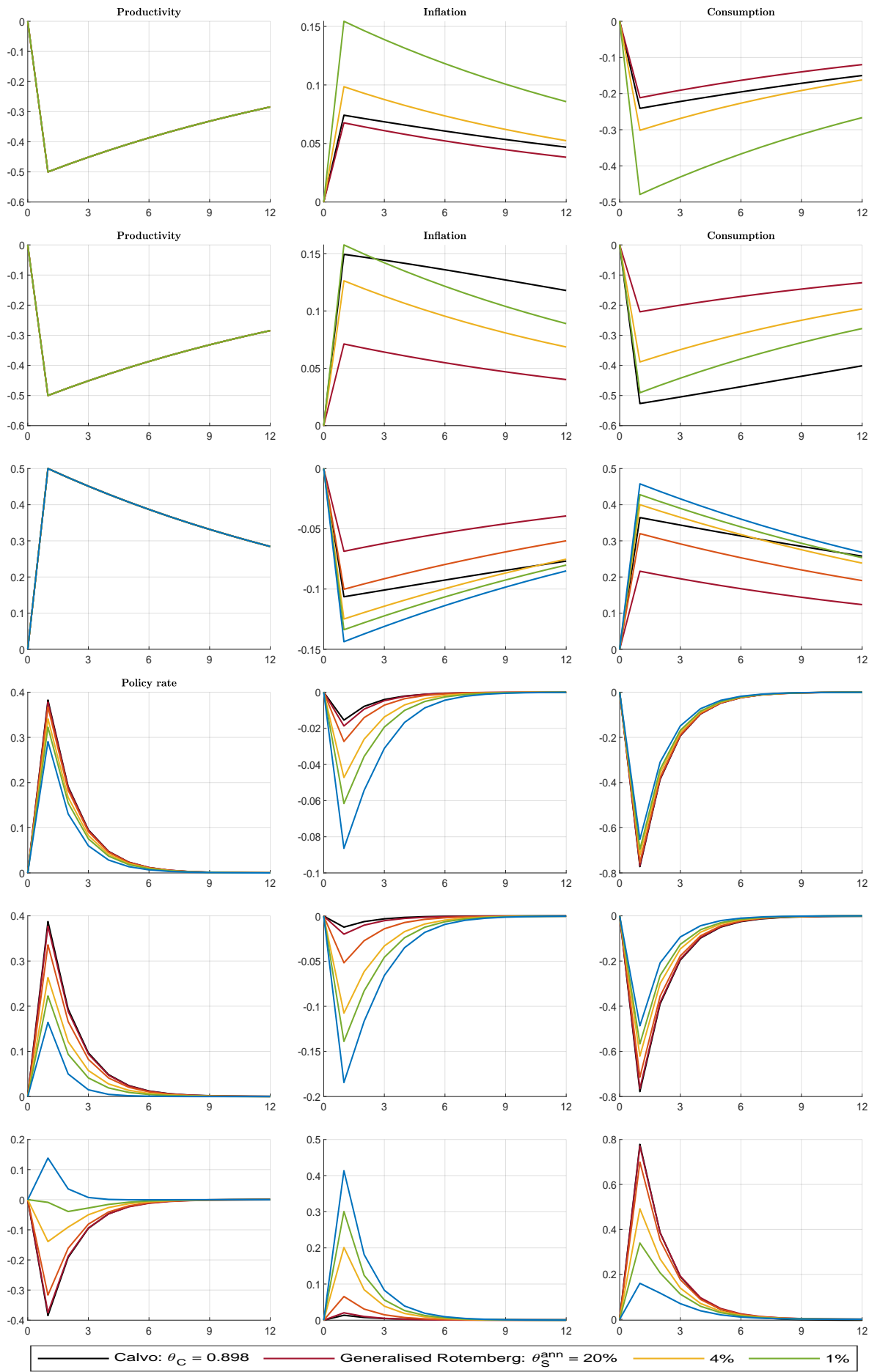


Figure C.1: Trend inflation and price flexibility when monetary policy is conducted by means of a Taylor rule

utility is very close. The terminal condition is set by assuming that after 120 periods (ten years), the value function of the firm is that of the new steady state with the persistently higher level of marginal costs. The path of industry price level P_t is then found by simple fixed point iteration.

Solving for the perfect-foresight path of the Rotemberg model of Section 4.2 is standard and can be done, for example, in Dynare.