Embodied Technical Change And The Fluctuations Of Unemployment And Wages*

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Abstract

The paper shows that a matching model where technological change is partially embodied in the job match is successful in explaining the variability of unemployment and vacancies. If we incorporate long-term wage contracts into the model, it also explains a number of stylized facts on the dynamics of real wages, which have been found in the empirical labor literature.

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1 Introduction

Shimer (2005) and Costain and Reiter (2003) have documented that the standard labor market matching model has problems in explaining the fluctuations of unemployment and vacancies. The purpose of this paper is to investigate whether these problems can be solved by adopting a somewhat different specification of technical progress. Standard RBC models, including the basic versions of the Mortensen/Pissarides model, assume that changes in aggregate productivity affect equally all firms and production processes. An alternative view with long tradition is that technical progress is embodied in new investment goods, and that only the new capital vintages enjoy the increase in productivity. In this paper I investigate the hypothesis that technical change is embodied, at least partially, in the match between a firm and a worker. This means that jobs created in a boom are more productive than jobs created in recessions, even after the boom is over. This hypothesis, if it is right, may be related to investment-specific technical change, as I will discuss in Section 4.4. But technical change embodied in matches may arise for reasons unrelated to physical investment. A boom can be seen as a time that offers many opportunities to firms. Many new ideas are around, new products and new markets are created, and firms strive to be the first in implementing ideas and conquer a share of the new markets. Even after entering a recession, where new ideas are scarce, the jobs that were created during good times keep a part of their productivity advantage.

Following most of the labor market matching literature, I will abstract in this paper from capital in production. To have the most parsimonious representation of embodied technical change, I assume that there is only one aggregate technology shock, and that labor productivity takes the following form:

$$Y(z, z_m) = (1 - \alpha_z)z + \alpha_z z_m, \qquad 0 \leq \alpha_z \leq 1 \tag{1}$$

Here, z is the current level of aggregate productivity, and z_m is the level of aggregate productivity prevailing at the time when the match was formed. The technological specification in standard RBC models is a special case of (1), where $\alpha_z = 0$. For $\alpha_z > 0$, technological change is partially "embodied" in the match.

It is difficult to test the specification (1) directly, because of composition bias in the workforce over the cycle, effects of changes in the capital stock and other issues. I think that the evidence reported in Bowlus (1995) provides some support for it. She finds that jobs that are created in recessions pay lower wages and dissolve more quickly than jobs created in booms. Moreover, "... the impact on tenure is greater for higher-educated workers and those employed in professional industries. This evidence suggests the cyclical phenomenon is one of general mismatching and not of workers taking stopgap jobs during recessions to get by." (Bowlus 1995, p.347). Nevertheless, there is at least one alternative potential explanation of Bowlus' findings, namely that the reduced labor market tightness in recessions creates a lower average match quality, as in Barlevy (2002). So far, I am not aware of any direct evidence on the validity of specification (1).

The aim of this paper is therefore to test (1) in an indirect way, by putting it into a standard labor market matching model, to see whether its implications for labor market aggregates and for real wages are supported by the data. A first step in this direction was done in Costain and Reiter (2003, Section 4.4), and it was found that embodied technical change can lead to a substantial increase in the variability of unemployment, therefore bringing the model closer to the data. There are two reasons for this. First, if technological progress is embodied in job matches, the observed labor productivity, being an average over the productivity of many past vintages, underestimates the productivity of new matches, which is what matters for vacancy creation and unemployment variation. Therefore, a correct calibration of a model with embodied technical change will imply a higher variability of current productivity. The second, more interesting effect is the following. If a new match is formed in a boom, the high productivity is partially embodied in the match, and will persist even if a recession comes. The possibility that a recession may arrive while the match continues, reduces the outside option of the worker, while the outside option of the firm is always zero. This tends to increase the fraction of the product of labor going to the firm, and hence the value of a match to the firm fluctuates more strongly over the cycle. This creates bigger fluctuations in vacancies, hiring and unemployment.

Costain and Reiter (2003, Section 4.4) identified two problems of using embodied technical change in a matching model. First, it turned out that this model creates a variability of wages much higher than what is found in the data. Second, if the embodied part of technological change is strong enough to explain the data, endogenous separations will occur. Whenever current aggregate productivity z is higher than the productivity when a match was created, z_m, the match has lower productivity than newly created matches, and it may be better for the firm and the worker to separate, and have the worker look for a new job. The evidence reported by Bowlus (1995) indicates that this does in fact happen. If it happens too often, the model generates dynamics that are clearly at odds with the aggregate data.

In the current paper, I address those problems using a recently developed version of the matching model with long-term wage contracts (Rudanko 2005). Those contracts dampen the fluctuations of wages of ongoing jobs, which helps to reconcile the model with the existing wage data. I slightly modify the model to allow for endogenous separations. To deal realistically with endogenous separations, we have to take into account that worker and firm, after a match has been formed, will probably undertake some match-specific investment, such as workers' training. Silva and Toledo (2007) have recently documented the costs of this type of investment, called "post-match labor turnover costs" (PMLTC), and investigated the consequences in a labor market matching model. In this paper, we will see that realistic PMLTC will sufficiently mitigate the problem of endogenous separations so as to make the model with embodied technical progress successful in matching labor market fluctuations.

After Costain and Reiter (2003), several other papers have included embodied technological change into matching models. It is present in Kennan (2005), although Brügemann (2005) shows that it does not drive the results in that paper. Independently of the present work, Eyigungor (2006) has developed a similar model with embodied technical change. In her model, employment relationships require match-specific capital, which assume the same role as my PMLTC to dampen endogenous fluctuations. Se does not comment on the implications of the model for wage dynamics. Hornstein, Krusell, and Violante (2007) embed labor market frictions into a model where technical progress is embodied in capital investment. They only analyze steady states.

The plan of the paper is as follows. The model is presented in Section 2. Section 3 derives some analytic results for a linear approximation of the model about the steady state. Numerical results for the model, calibrated to US data, are presented in Section 4. Section 5 concludes. Details of the computational procedures are given in an appendix that can be downloaded from "http://www.econ.upf.es/∼reiter/research.html".

2 The Model

The economy is populated by a unit mass of workers. Workers order consumption streams according to the utility function $\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i U(c_{t+i})$. The concavity of U implies risk aversion, but we assume that workers do not have access to the capital market; they can neither borrow nor save.[1] Therefore, their consumption equals their wage w_t while employed, and the constant b while unemployed. We will refer to b as "unemployment benefits", but following most of the matching literature, we effectively treat it like home production. This means in particular that we do not take into account that b has to be financed by a government through taxes etc.

A firm is a filled job. It employs one worker and produces output according to the production function (1). In each period, the firm becomes permanently unproductive with probability δ. In this case, the match dissolves and the worker becomes unemployed. Whenever a match separates, either endogenously or exogenously, the vacancy (the firm) disappears.We assume that firms are owned by risk neutral entrepreneurs, but we effectively ignore the entrepreneurs in the analysis of the model (they consume profits in every period, and we do not worry about their consumption being positive or not). The asymmetry

[1]This assumption, in particular the part that households cannot save, is overly strong and is only made to keep the contracting problem of workers and firms tractable. Beaudry and Pages (2001) give a calibration where it is an equilibrium outcome that workers do not save.

between risk-neutral entrepreneurs and risk averse workers provides an incentive to long-term wage contracting where firms insure workers against wage fluctuations.

2.1 Matching Technology

The measure of unemployed workers is denoted by U, and the measure of open vacancies by V. New jobs are created according to the matching function

$$M = A_m U^{\alpha} V^{1-\alpha} \tag{2}$$

We define labor market tightness as $\theta = \frac{V}{U}$. Then we can write the probability of a firm to fill the vacancy as $p^F = \frac{M}{V} = A_m \theta^{-\alpha}$ and the probability of a worker to find a job as $p^W = \frac{M}{U} = \theta p^F = A_m \theta^{1-\alpha}$.

2.2 Model With Short-Term Wage Contracts

We first consider a version of the model that is as close as possible to the standard Mortensen/Pissarides model. We assume that households have linear utility, and that wages are renegotiated every period. The only new element is the technology specification (1).

Denote by $F(z, z_m)$ the value of a filled job to the firm, and by $V(z, z_m)$ the value of a job to the worker. They are both a function of the current state of productivity z and the productivity of the time when the match was formed, z_m. The value of being unemployed is denoted by $V^u(z)$ and depends only on current technology. The value functions satisfy

$$V(z, z_m) = w(z, z_m) + \beta \, \mathrm{E}_{z'} \left[(1-\delta) V(z', z_m) + \delta V^u(z') \right] \tag{3a}$$

$$V^u(z) = b + \beta \, \mathrm{E}_{z'} \left[p^W(z) (V(z', z') + (1 - p^W(z)) V^u(z')) \right] \tag{3b}$$

$$F(z, z_m) = Y(z, z_m) - w(z, z_m) + \beta (1-\delta) \, \mathrm{E}_{z'} F(z', z_m) \tag{3c}$$

Each period, the wage is set so as to satisfy the Nash bargaining conditions

$$\alpha F(z, z_m) = (1-\alpha) \left(V(z, z_m) - V^u(z) \right) \tag{4}$$

We impose the Hosios condition, saying that the workers' bargaining share α equals the elasticity of matches with respect to unemployment in (2). We use in (4) that the outside option of the firm is always zero. This is because vacancies disappear after a match is separated, and new vacancies can be freely created such that the following zero profit condition holds:

$$\kappa = p^F(z)F(z, z) \tag{5}$$

In this version of the model, we do not allow for endogenous separation. The firm and the worker continue the match even if the joint surplus is negative, until they are hit by the separation shock with probability δ. One can interpret this model as describing the case of small fluctuations, in which case the surplus is always positive. Endogenous separations will be handled in the next subsection.

2.3 Model With Long-Term Wage Contracts

We will see in Section 4 that the model with continuous re-bargaining creates excessive volatility of wages compared to the data. This problem is the more severe the bigger is the embodied part of technological change. We therefore investigate whether a model with long-term wage contracts can bring the model in line with the data.

The model I use here is very similar to Rudanko (2005), except for assuming embodied technical change. We now assume that worker and firm sign a wage contract at the beginning of the employment relationship. A contract that is signed in period t specifies a sequence of wage payments $w_{t+i}(z^{t+i})$, $i = 0, 1, \ldots$. Each wage is conditional on the history of shocks $z^{t,t+i} = (z_t, z_{t+1}, \ldots, z_{t+i})$. No wage payments can be made after the separation of the match.

Worker and firm choose a contract that is privately efficient (that means, it is impossible to make both firm and worker better off). The set of efficient contracts can be characterized by the firm's expected profit function $F(v, z, z_m; V^u(.))$. It is a function of the current state of technology z, the technology at the time the match was formed, z_m, and the expected value that the firm has promised to the worker, v. We treat v here as a choice parameter of the firm; the higher v, the lower will be the profit that the firm can earn from the employment relationship. The firm's profit function also depends on the outside option of the worker in the different states of nature, $V^u(.)$. Note that, once firm and worker have separated, the only relevant state variable is current technology z. The outside option is therefore a function of z only, $V^u(z)$. For notational simplicity, we suppress this argument and simply write $F(v, z, z_m)$.

We now write down the functional equations that characterize the firms' profit function under limited commitment. Whenever the value promised to the worker v exceeds her outside option, the match must be continued, and then the firm's profit function satisfies

$$F(v, z, z_m) = \max_{w, V(z')} \{Y(z, z_m) - w + \beta\,\mathrm{E}_{z'}(1-\delta)F(V(z', z_m), z', z_m)\}$$
$$\text{for } v > V^u(z) \quad \text{(6a)}$$

In the case where the promised value v is not greater than the outside option of the worker $V^u(z)$, the firm is free to endogenously separate the match, which gives $V^u(z)$ to the worker and zero to the firm. This can be the optimal choice if $z > z_m$. Then

$$F(v, z, z_m) = \max\{\max_{w, V(z')} \{Y(z, z_m) - w + \beta\,\mathrm{E}_{z'}(1-\delta)F(V(z', z_m), z', z_m)\}, 0\}$$
$$\text{for } v <= V^u(z) \quad \text{(6b)}$$

The maximizations in (6a) and (6b) are subject to the constraints

$$v = u(w) + \beta\,\mathrm{E}_{z'}[(1-\delta)V(z', z_m) + \delta V^u(z')] \quad \text{(6c)}$$

$$V(z', z_m) \geq V^u(z'), \qquad \forall z' \quad \text{(6d)}$$

$$F(V(z', z_m), z', z_m) \geq 0, \qquad \forall z' \quad \text{(6e)}$$

The inequality (6d) is the condition that the worker wants to continue in the match. Inequality (6e) is the analogue for the firm. Notice that the feasible set of promised values is nonempty, because the firm can always promise to separate, which satisfies both (6d) and (6e).

Properties of these optimal contracts have been derived by Thomas and Worrall (1988) and Rudanko (2005, Section 2). In particular, the wage stays constants over the match as long as no participation constraint binds. If, for example, the constraint (6d) starts binding, the firm has to increase the wage to keep the worker within the firm. In the full-commitment case, the wage is constant over the life of the match.

It remains to determine which of the contracts on the efficiency frontier is chosen. We assume that, when firms and workers are matched in the aggregate state z, they chose the optimal contract with the entry value $V^e(z)$ that maximizes the Nash product[2]

$$V^e(z) = \underset{v}{\operatorname{argmax}} \left(v - V^u(z)\right)^{\alpha} \left(F(v, z, z) - \kappa_M\right)^{1-\alpha} \tag{7}$$

where κ_M denotes the PMLTC motivated in the introduction. We assume that κ_M is paid by the firm after the match is formed. $F(v, z, z)$ has to be understood as the firm value after paying κ_M.

Using the envelope condition

$$\frac{\partial F\left(V^e(z), z, z\right)}{\partial V} = -\frac{1}{u'(w(z,z))} \tag{8}$$

and denoting by $F^e(z)$ the firm's value at $\left(V^e(z), z, z\right)$, we can write the first order condition for (7) as

$$\alpha(F^e(z) - \kappa_M)u'(w_e(z)) = (1-\alpha)\left(V^e(z) - V^u(z)\right) \tag{9}$$

which is analogous to (4). Vacancy creation by firms is governed by the zero-profit condition

$$p^F(\theta(z))(F^e(z) - \kappa_M) = \kappa \tag{10}$$

To close the model, we have to determine the outside option of the worker which enters the constraint (6d). It is given by

$$V^u(z) = u(b) + \beta \, \mathrm{E}_{z'}\left[V^u(z') + p^W(z')\left(V^e(z') - V^u(z')\right)\right] \tag{11}$$

[2]Rudanko (2005, Prop. 2.4) starts from a framework of competitive search and shows that the equilibrium is the same as with Nash bargaining where the bargaining weight of the worker equals the elasticity of matches w.r.t. unemployment. For our purposes, we can start right away with the Nash bargaining framework.

3 The Effects of Embodied Technical Change

Before presenting numerical results of the model, we analyze the effects of the technology specification (1) by a linearization of the model with linear utility about the deterministic steady state. Notice that endogenous separations are not an issue here, because implicitly we are dealing with infinitesimal fluctuations about the steady state. For the linearization, we assume that z follows an AR(1) process, where the parameter σ is the speed at which the process returns to its mean. In Proposition 1 we state a result about the response of the job finding probability to aggregate productivity. For this we assume that, at the time of the match, the total expected surplus is shared according to the Nash formula, but we need not take a stand on whether the wage is renegotiated in later periods.

Proposition 1. *In the model with linear utility, to a first order approximation about the steady state, the elasticity $\eta_z^{p^W}$ of the job finding probability with respect to aggregate productivity z satisfies*

$$\eta_z^{p^W} = \frac{1-\alpha}{\alpha} \frac{1+\beta(1-\delta)\sigma\frac{\alpha_z}{1-\beta(1-\delta)}}{1-\beta\left[(1-\delta)(1-\sigma)-p_W^*(1-\alpha\sigma)\right]} \frac{1-\beta(1-\delta-\alpha p_W^*)}{1-b} \tag{12}$$

$$\approx \frac{1-\alpha}{\alpha} \frac{r+\delta+\alpha p_W^*}{r+\delta+\sigma+p_W^*} \frac{r+\delta+\sigma\alpha_z}{r+\delta} \frac{1}{1-b} \tag{13}$$

Proof. See Appendix A. □

The approximation (13) gives the continuous time limit, to which our model is quite close, since we solve it with weekly frequency. Here, r denotes the interest rate, related to the time discount factor by $\beta = 1/(1+r)$.

Inspecting (13), we first learn that the responsiveness of the job finding rate is inversely related to $1-b$, the difference between labor productivity and unemployment benefits. This has been pointed out and explained in Costain and Reiter (2003). The new insight is that the elasticity is increasing in α_z, the degree of embodiedness. Moreover, this effect is stronger if σ is higher. From the introduction we know that the firm's value fluctuates more under embodied technical progress, because the firm can lock in the currently high level of productivity. Obviously, this effect is more important if productivity returns faster to its mean. In the limit case $\sigma = 0$, productivity changes are permanent, and $\eta_z^{p^W}$ is unaffected by α_z. If the model is calibrated to US data, we have $p_W^* >> r + \delta + \sigma$. In that case, (13) can be further simplified to

$$\eta_z^{p^W} \approx (1-\alpha)\frac{r+\delta+\sigma\alpha_z}{r+\delta}\frac{1}{1-b} \tag{14}$$

Equ. (14) makes clear how embodied technical changes affects the elasticity. What matters is the expected change in z, measured by σ, which affects the future outside option of the worker, compared to the effective expected lifetime of a job, measured by $r + \delta$.

If we further assume that wages are renegotiated each period according to (4), we can derive the reaction of wages to aggregate productivity:

Proposition 2. *In the model with linear utility and continuous Nash bargaining, to a first order approximation about the steady state, wages satisfies*

$$\frac{d\,w(z,z_m)}{d\,z} = \alpha(1-\alpha_z) + \beta\alpha(1-(1-\alpha)\sigma)\eta_z^{p^W}\frac{(1-b)p_W^*}{1-\beta(1-\delta-\alpha p_W^*)} \tag{15}$$

$$\approx \alpha(1-\alpha_z) + \alpha\eta_z^{p^W}(1-b)\frac{p_W^*}{r+\delta+\alpha p_W^*} \tag{16}$$

$$\frac{d\,w(z,z_m)}{d\,z_m} = \alpha\alpha_z \tag{17}$$

Proof. See Appendix A. □

Notice that $\frac{d\,w_e(z)}{d\,z} = \frac{d\,w(z,z_m)}{d\,z} + \frac{d\,w(z,z_m)}{d\,z_m}$. If we use again the fact that in a calibration to US data we have $p_W^* >> r + \delta$, (16) can be even further simplified to

$$\frac{d\,w(z,z_m)}{d\,z} \approx \alpha + \eta_z^{p^W}(1-b) \tag{18}$$

If $(1 - b)$ is small, wages cannot fluctuate very strongly because they are closely tied to productivity. If the elasticity $\eta_z^{p^W}$ is high, wages tend to fluctuate a lot, because the changes in job finding probability make the outside option of the worker fluctuate. In fact we will see in Section 4.3 that the model generates excessive wage volatility compared to the data. I think this is not a feature specific to embodied technical progress; it will show up in any model that has flexible wages, a sizeable match surplus, and a strongly fluctuating job finding probability. Existing specifications of the matching model have moderate wage fluctuations because either $\eta_z^{p^W}$ is small (Shimer 2005) or $1 - b$ is small (Hagedorn and Manovskii 2005) or the real wage is assumed to be rigid (Hall 2005).

4 Numerical Results

4.1 Parameter values

The model period is 1/48 of a year, corresponding roughly to a week. For the parameters I use standard values from the literature. The discount rate is set to 1.2% quarterly, so $\beta = 0.988^{1/14}$. In the model with long-run contracts, I use log utility, $U(c) = \log x$. The parameter b, which captures both unemployment benefits and the value of leisure, is the key parameter that determines the volatility of tightness and unemployment. I use $b = 0.745 - \kappa_M(1 - \beta(1 - \delta))$. For $\kappa_M = 0$, this gives $b = 0.745$, which is taken from Costain and Reiter (2003, Table 1), so as to give a realistic response of the model to long-run changes in unemployment benefits and taxation. For $\kappa_M > 0$, b gets reduced so as to keep the total discounted surplus of the match unchanged. Without such an adjustment, κ_M would reduce the total surplus of the match and increase the variability of unemployment, as explained in Costain and Reiter (2003).

For the elasticity of matches w.r.t. unemployment I use $\alpha = 0.4$. This is well within the range of values that Petrongolo and Pissarides (2001, Table 3) report. With this value, the model strikes a balance between explaining the variability of unemployment and the variability of tightness. If I use a higher value of α, as Shimer (2005) and Rudanko (2005) do, the model tends to underestimate the variability of unemployment, and overestimate that of tightness. In this sense, the parameter is "estimated" from the data. For the job separation rate, some recent papers have used a value of 40% annually. Here I deviate and rather use 25% annually, $\delta = 0.25/48$, which is closer to what earlier papers in the matching literature have used. The main reason is that I assume in the model that technical progress is linked to the match, not to the job, while in reality it is probably a mixture of both. Using a separation rate of 0.4 dilutes the effect of embodied technical change too much. The matching efficiency A_m is normalized to unity; this parameter only scales the absolute number of vacancies, which is irrelevant for us. In each experiment, the vacancy cost parameter is set such that the steady state unemployment rate is 5.67 percent, the average in the US in the period 1951–2003.

The parameter α_z, which measures the degree of embodied technical progress, is crucial to obtain the right variability of unemployment. Next to the case $\alpha_z = 0$, I will present results for two different values of α_z that are chosen so as to match important cyclical characteristics of the data. Choosing $\alpha_z = 0.576$, the model generates the same variance of unemployment as in the data. Mortensen and Nagypal (2005, p.8) and Rudanko (2005, p.21) suggest that the model should only explain the part of the volatility in the data that is related to the movements of aggregate labor productivity, more precisely the regression coefficient of unemployment on average labor productivity. This is achieved by choosing $\alpha_z = 0.302$. That there are values of α_z that achieve those targets is an important success of the model.

For the productivity process, I use a 9-state Markov chain. From each state, only neighbouring states can be reached within one model period. The transition probabilities where chosen such that in each state, the conditional expectation satisfies $\mathrm{E}_t\, z_{t+1} = \rho_z z_t$, and the conditional standard deviation is σ_z. For each value of α_z, I choose σ_z and ρ_z so as to match the unconditional variance and the quarterly autocorrelation of average labor productivity (which is not equal to z in the case $\alpha_z > 0$). Since the model with long-term contracts takes a long time to compute, the values of σ_z and ρ_z were calibrated for the model with short-term contracts. I use the same values then with long-term contracts, which makes little difference for the calibrated targets. The states of the Markov chain where chosen such that the outer points are ± 2.5 times the unconditional standard deviation of z.

For the PMLTC, I follow the calibration in Silva and Toledo (2007). The two most important types of costs are the training costs and the initial productivity gap of a newly hired worker. Total average cost of man-hours of training are estimated as 55 percent of the quarterly wage of newly hired workers (Silva and Toledo 2007, page 5). The productivity of a newly hired worker is 40 percent lower in the beginning, and this gap is on average closed within one year (Silva and Toledo 2007, page 10). Based on these numbers, I consider PMLTC of 6 weeks of prodution ($\kappa_M = 6$) as a conservative estimate, while 12 weeks of production ($\kappa_M = 12$) is roughly equivalent to the calibration of Silva and Toledo (2007). I will show results for both calibrations, next to the case of $\kappa_M = 0$. In my model, the PMLTC have to be paid at the beginning of the match, while in Silva and Toledo (2007) they accrue over a more extended period of time (training and productivity gap), but this should not affect much the dynamics of the model.

4.2 Explaining the cyclicality of labor market aggregates

Table 1 reports statistics for detrended US labor market data from 1951-2003. In Table 2 we find the simulation results from the model with short-term contracts, as described in Section 2.2. In Table 3 are the results from the model with long-term contracts, described in Section 2.3. Simulation results are averages over 500 runs, each of 63 years, with the first 10 years discarded, so that the length of the simulated series is 53 years as in the data. Data and simulation results are detrended by the HP filter with smoothing parameter 10^5.

The first block of Table 2 gives the result of the standard Mortensen/Pissardes model (the case $\alpha = 0$). It documents the wellknown failure of this model to explain the variability of unemployment, vacancies, tightness and the job finding probability. The model falls short of explaining the data by a factor of almost 5. Less dramatic shortcomings of the model are the too low autocorrelation of vacancies, and that the Beveridge curve, the negative correlation between unemployment and vacancies, is not as strong in the model as in the data. From the first block of Table 3 we see that the introduction of log-utility and long-term contracts has very little effect on the dynamics of unemployment and tightness. This is in line with the findings of Rudanko (2005).

The second block of Tables 2 and 3 shows what happens if labor productivity is partially embodied. The value $\alpha_z = 0.302$ was chosen so as to match the elasticity of unemployment with respect to labor productivity. We see that the corresponding elasticities of vacancies, tightness and the job finding probability is also in line with the data. The autocorrelation of vacancies and the Beveridge curve vary little in response to the change in α_z. What is interesting in Table 3 is that the endogenous separations, which are documented in the variability of the separation rate δ, are infrequent enough not to affect the dynamics of the main labor market aggregates. Note that in this calibration, I still assume $\kappa_M = 0$.

In the last block of Table 2, we set $\alpha_z = 0.576$ so as to match the variability of unemployment. The standard deviations of vacancies, tightness and the job finding probability are also approxiately right. The autocorrelation of vacancies and the Beveridge curve change slightly, but in the wrong direction. If we compare this to the results in the third block of Table 3, we see that now the endogenous separation are so strong that the Beveridge curve gets the wrong sign. This is because in times of high productivity, there is massive endogenous separation, leading to temporary spikes in unemployment. If there is no reasonable way to avoid these waves of endogenous separations, we have to conclude that the model with a high degree of embodiedness is incompatible with the facts. In the fourth and fifth block of Table 3, we therefore investigate whether realistically sized PMLTC are able to bring the model in line with the data. We see that already the conservative calibration $\kappa_M = 6$ goes a long way in solving the problem. The Beveridge curve is less strong than in the model with exogenous separations only. Interestingly, the standard deviation of the separation rate is close to what we find in the data. That separations are positively correlated with labor productivity is counterfactual, but one should remember that in the data, the countercyclicality of separations is rather weak. The endogenous separation mechanism in this model could be part of the explanation.

In the calibration $\kappa_M = 12$, we see that endogenous separations are again infrequent, and the labor market dynamics of the model differs very little from the model with short-term contracts.

In sum, if we want to use the matching model to explain the systematic part of unemployment and vacancies (the elasticity with respect to aggregate labor productivity z), we can achieve this with a low level of embodiedness, $\alpha_z = 0.302$. In that case, endogenous separations are infrequent even without assuming any PMLTC. If we want the model to generate unconditional variances of unemployment and vacancies as high as in the data, a higher level of embodiedness is necessary, $\alpha_z = 0.576$. Not allowing for any PMLTC, the implications of the model for the correlation of unemployment and labor productivity are clearly counterfactual. However, PMLTC of the size proposed by Silva and Toledo (2007) brings the model predictions in line with the data.

4.3 Explaining the cyclicality of real wages

The relationship of wages and the business cycle is difficult to pin down. From Table 1 we see that the average wage (hourly compensation nonfarm business sector) is about as volatile as labor productivity per worker, but is only mildly procyclical. Relating hourly compensation to the hourly labor product (not product per worker as in Table 1) in the nonfarm business sector, after taking logs and detrending, one finds that the correlation coefficient is 0.5173 for the time period 1951-2003. The regression coefficient of wages on the hourly product is 0.573. Measuring cyclicality by the correlation with unemployment, we come to similar conclusions. The correlation coefficient between hourly compensation and the unemployment rate (detrended, but no logs) is only -0.096, and the regression coefficient is -0.160. This means that a one percentage point increase in unemployment is related to a 0.160 percent reduction in wages. Based on this finding, some recent papers (Hall (2005), among others) have argued that the low responsiveness of real wages to aggregate productivity is a key element in explaining the high variability of unemployment.

Studies using panel data tend to find a stronger procyclicality of wages, partly because they can control for the composition bias in the workforce over the cycle (Solon, Barsky, and Parker 1994). This finding has been reinforced by recent studies which distinguish the cyclicality of job stayers and job movers. Hart (2003) and Devereux and Hart (2005) for UK data, and many studies for US data (cf. Shin and Solon (2004) and references there) find that the wages of workers when they change jobs are significantly more flexible than the wages of those who stay in the same job (wages of workers who change the job but stay in the same firm are somewhere in-between). The results summarized in Table 4 find that a one percentage point increase in the unemployment rate is associated with a reduction in the real wage of male job stayers of 1–2 percent, and of male job movers of 2–3 percent. In yet preliminary work based on CPS data, Haefke, Sonntag, and van Rens (2006) report the elasticity of wages with respect to aggregate labor productivity. For job movers, they find an elasticity of about 1, while for job stayers, they find an elasticity of about 0. Again, the finding is that entry wages, which determine the hiring incentives of firms, are flexible, while the wages of incumbents move relatively little.

From Table 2 we see that the model with short-term contracts is unable to explain those stylized facts. The model predicts almost perfectly cyclical wages. With disembodied technical change, the wage fluctuates much more than in the data, for the reasons analyzed in Section 3. An interesting finding is that embodied technical progress does not generate enough difference between average and entry wages. This model is therefore at odds both with the macro and the micro evidence on wages.

The model with long-term contracts fits the data better. The fluctuations of average wages are close to what we find in aggregate data, and the model predicts a much weaker correlation between average wages and producitivity or unemployment, although still somewhat higher than in aggregate data (cf. Table 3). Notice that the model with long-term contracts and neutral technical change (α_z) predicts too low wage variability, while the model with short-term contracts and embodied technical change predicts too much of it. Long-term contracts and embodied technical change gets the variability about right. Table 5 reports the results from a regression of changes in wages on the change in average productivity and on the change in unemployment. This should be compared to the empirical results in Table 4.[3] We learn that the model with long-term contracts generates a sharp distinction between average and entry wages, even stronger than in the data. These results do not depend very much on the parameter α_z, therefore they do not

[3] A referee has pointed out that empirical estimates and model simulations are not fully comparable, since the model talks about movements from unemployment to employment, while the movers in the data also include job-to-job transitions, which probably have a different wage dynamics.

provide independent evidence on whether technical change is embodied or not.

Four main conclusions emerge from these results. First, the model with long-term contracts avoids the excessive wage volality caused by embodied technical progress. The variability of aggregate wages is similar in the model and in the data. Second, there is little evidence of exogenous wage rigidity. The cylicality of wages for *new hires*, which determine the firms' incentives to create vacancies, is similar in the micro data and in the model. The model is not perfect in this respect: the elasticity of wages of new hires both with respect to aggregate labor productivity and with respect to the unemployment rate is somewhat higher in the model than in the data. This might be due to the fact that neutral and embodied technical change are perfectly correlated, due to the one-shock specification (1). Future work should relax this assumption. Third, the results give additional support for the importance of long-term wage contracts. Although embodied technical progress as specified in (1) is a potential explanation for why entry wages fluctuate more than average wages, the quantitative analysis reveals that this effect is not strong enough. We need long-term contracts, which stabilize the wages on existing relationships through wage insurance.[4] Finally, the model underpredicts the cyclicality of the wages of job stayers. Future work might solve this problem by a model where workers are allowed to save, which reduces the wage-smoothing motive in the optimal contract.

[4]A related strand of literature identifies cohort effects in wages, and find that the time at which a worker enters a firm or enters the labor market for the first time, has a persistent effect on a wage. Workers who entered in a recession will receive lower wages than workers who entered in a boom, even many years after entering the firm (see for example Beaudry and DiNardo (1991), Baker, Gibbs, and Holmstrom (1994), Oreopoulos, von Wachter, and Heisz (2006)). Both long-term contracts and embodied technical change help to explain these stylized facts. Again it turns out that long-term contracts are needed to explain the magnitude of the effects in the data. More details can be found in an earlier version of this paper, Reiter (2006).

4.4 Investment-Specific Technological Change and the Labor Market

In Section 4.2 we have seen that a substantial degree of embodied technical progress is necessary to explain the variability of unemployment. As explained in the introduction, there seems to be no direct evidence that would tell us to what extent technical progress is embodied in job matches. There is, however, ample evidence on investment-specific technological change. If new capital is more productive than existing vintages, and if new job matches get a higher share of new capital than the already existing matches, than this may explain productivity differentials between jobs of different vintages. In Hornstein, Krusell, and Violante (2007, Section 2), this is achieved by assuming that one worker is always matched with one unit of new capital, and that capital becomes firm-specific, once it has been installed.

Our model does not include capital, but we can nevertheless ask whether the existing measures of investment-specific technological change, if taken as a proxy for vintage-specific productivity, are helpful in explaining labor market aggregates. For that purpose I consider the Cummins and Violante (2002) measure of the technological gap for equipment and software (CVG). It is defined as the percentage difference in the efficiency of a new capital vintage vs. the average efficiency of old vintages (where capital is measured in constant-quality consumption units). Using the same measure, Fisher (2006) finds that investment-specific technology shocks have an important role in explaining business cycle fluctuations.

While the low-frequency variations of the CVG are most striking, it also fluctuates a lot at business cycle frequencies. The following statistics as well as Figure 1 refer not to the original CVG, but to $\log(1 + CVG)$, to make the series compatible with the theoretical counterpart in the model simulations. Then the CGV has a standard deviation of 0.0164 after HP filtering with smoothing parameter 100, and of 0.0249 after filtering with parameter $10^5/16$. Note that I divided the usual smoothing parameters by 16, since CGV has annual frequency. For comparison, the technological gap in our model has standard deviation 0.007 in the case of $\alpha_z = 0.302$, and 0.021 in the case of $\alpha_z = 0.576$.

Figure 1 plots the log of the job finding probability p^W, together with the CGV, scaled by a factor of $0.36 \cdot 9.9287$. The factor 0.36 is motivated by the assumption that the share of capital in production is 0.36, and 9.9287 is the regression coefficient of $\log(p^W)$ on the labor productivity gap in the model with $\alpha_z = 0.576$. The dotted line in Figure 1 can be interpreted as the job finding probability as predicted by the movement in CGV. Both series are demeaned, but not detrended, because the CVG seems to have structural breaks and is difficult to detrend. We see that from about 1960 to 1985, the predicted value follows the data reasonably well. Outside this range, we are less successful. For example, the increase in p^W during the Korean war and the fall in p^W during the 1990/91 recession seem to be unrelated to changes in CVG. Consistent with this finding, Canova, Lopez-Salido, and Michelacci (2007) find that the recession of the early 90's is almost entirely explained by neutral rather then investment-specific technology shocks. Obviously, the one-shock specification (1) is not rich enough to explain all the cyclical episodes of the post-war US economy.

5 Conclusions

The paper has shown that a slight modification of the standard specification of labor productivity, which allows for partial embodiment of technical progress in a match, goes a long way in reconciling the Mortensen/Pissarides matching model with the US data. Without resorting to any exogenous rigidities, the model can explain the high variability of vacancies, job finding probabilities and the unemployment rate. The version with long-term wage contracting is consistent with the low volatility of the average wage, and the fact that wages of newly formed matches fluctuate more than average wages.

The question is then whether our description of the productivity process, Equ. (1), is realistic. That this process helps the matching model to explain labor market fluctuations is certainly indirect evidence in its favor. It is encouraging that the model explains reasonably well the US labor market fluctuations in the period 1960 to 1985, if we take investment-specific technical change as a proxy for embodied technical change. To what extent the evidence reported in Bowlus (1995) can be seen as supportive, will finally depend on the success of alternative explanations such as Barlevy (2002). Unfortunately, the dynamics of wages does not tell us very much about productivity. Once we assume long-term contracts, embodied technical progress does not make sufficient difference in the dynamics of entry or average wages to provide clear evidence in favor or against embodied technical change.

Future research should address these issues. On the theoretical side, one should explore structural explanations of why technical progress, through innovations in capital and consumption goods, is embodied in job matches. This might guide more empirical studies to find direct evidence for embodied technical change. A satisfactory model would allow productivity to be embodied in jobs such that after a separation, a valuable vacancy might be left. This would require a substantially more complicated model than the one considered in this paper.

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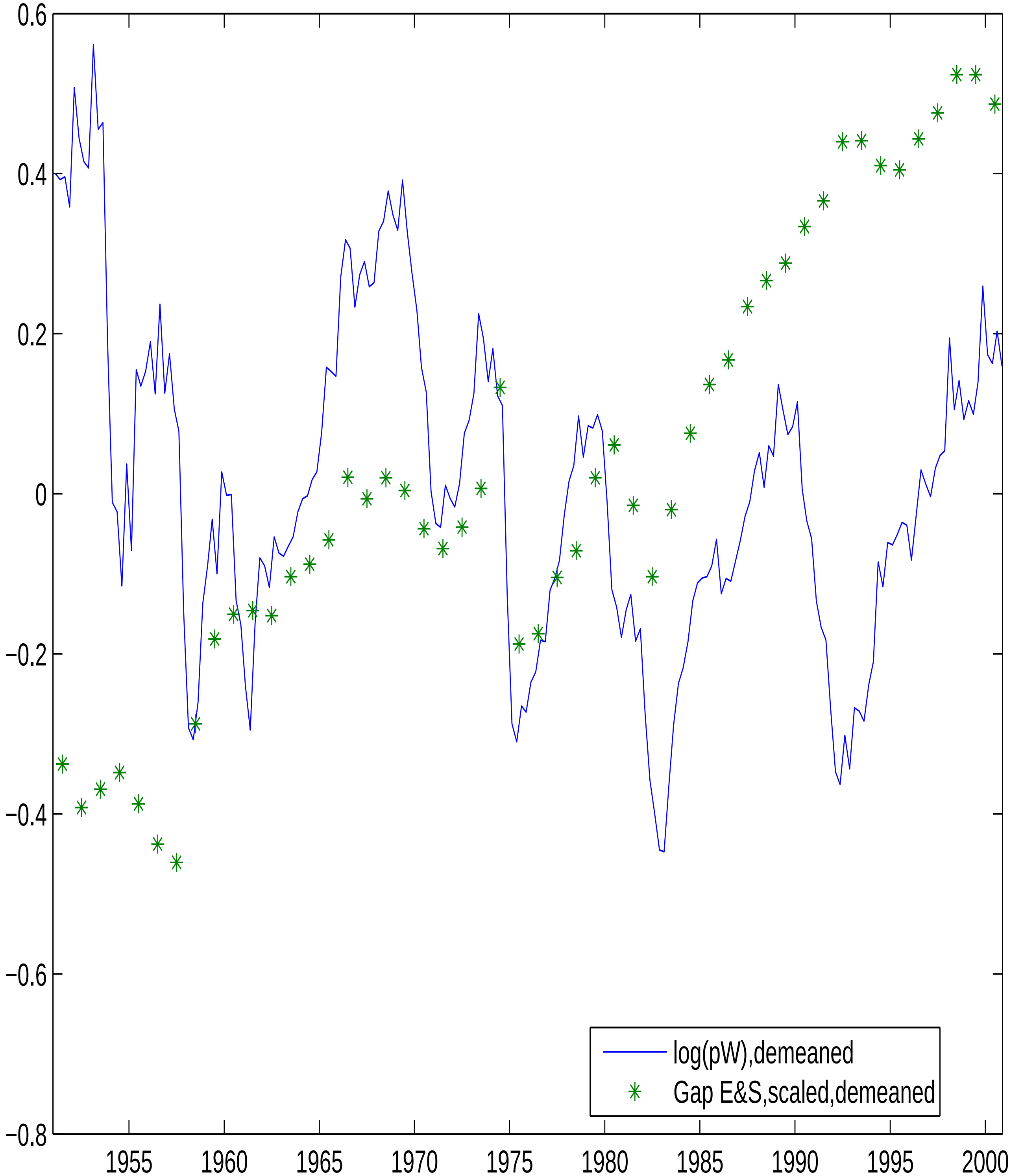

Figure 1: Technological gap and job finding probability, US data

Table 1: Labor market fluctuations, US data 1951-2003

	z^a	U	Vac	θ	p^W	δ	W^a
stdev	0.020	0.191	0.202	0.383	0.164	0.067	0.018
relstd	1.000	9.482	10.053	19.045	8.168	3.320	0.895
autocorr	0.889	0.939	0.948	0.946	0.912	0.635	0.941
corr(u)	-0.417	1.000	-0.901	-0.973	-0.959	0.603	-0.058
corr(z)	1.000	-0.417	0.383	0.410	0.402	-0.570	0.350
elast(z)	1.000	-3.958	3.852	7.810	3.288	-1.891	0.313

Note: all data are in logs and detrended by the HP filter with smoothing parameter 10^5. z^a: average labor productivity; U: unemployment rate; Vac: vacancies; θ: tightness; p^W: job finding probability; δ: separation rate; W^a: average hourly wages non-farm business sector

Table 2: Simulation results, model with short-term contracts

	z^a	U	Vac	θ	p^W	δ	W^a	W^e	z	z/z^a
Short-term contracts, $\alpha_z = 0$										
stdev	0.020	0.044	0.042	0.083	0.050	0.000	0.019	0.019	0.020	0.000
relstd	1.000	2.196	2.101	4.090	2.454	0.000	0.944	0.944	1.000	0.000
autocorr	0.887	0.937	0.759	0.887	0.887	0.000	0.887	0.887	0.887	0.000
corr(u)	-0.953	1.000	-0.814	-0.954	-0.954	0.000	-0.953	-0.953	-0.953	0.000
corr(z)	1.000	-0.953	0.948	0.999	0.999	0.000	1.000	1.000	1.000	0.000
elast(z)	1.000	-2.093	1.991	4.084	2.451	0.000	0.944	0.944	1.000	0.000
Short-term contracts, $\alpha_z = 0.302$										
stdev	0.020	0.084	0.082	0.158	0.095	0.000	0.028	0.031	0.027	0.008
relstd	1.000	4.132	4.085	7.778	4.669	0.000	1.401	1.542	1.338	0.374
autocorr	0.888	0.930	0.738	0.874	0.874	0.000	0.878	0.875	0.875	0.857
corr(u)	-0.958	1.000	-0.793	-0.947	-0.947	0.000	-0.950	-0.945	-0.945	-0.819
corr(z)	1.000	-0.958	0.907	0.985	0.985	0.000	0.995	0.990	0.990	0.868
elast(z)	1.000	-3.960	3.705	7.664	4.600	0.000	1.394	1.528	1.326	0.326
Short-term contracts, $\alpha_z = 0.576$										
stdev	0.020	0.189	0.212	0.373	0.225	0.000	0.051	0.059	0.039	0.021
relstd	1.000	9.482	10.665	18.717	11.266	0.000	2.534	2.940	1.941	1.057
autocorr	0.889	0.916	0.686	0.839	0.839	0.000	0.851	0.845	0.845	0.829
corr(u)	-0.927	1.000	-0.729	-0.921	-0.921	0.000	-0.922	-0.915	-0.919	-0.810
corr(z)	1.000	-0.927	0.785	0.916	0.916	0.000	0.958	0.941	0.941	0.780
elast(z)	1.000	-8.786	8.365	17.137	10.314	0.000	2.428	2.767	1.827	0.827

Notes: W^e: wages of new matches; z: current aggregate labor productivity (equal to productivity of new matches)
stdev: unconditional standard deviation in simulation; relstd: stdev(x) / stdev(z^a); autocorr: first order autocorrelation; corr(u): correlation with U; corr(z): correlation with z^a; elast(z): regression coefficient on z^a (equals product of relstd and corr(z))

Table 3: Simulation results, model with long-term contracts

	z^a	U	Vac	θ	p^W	δ	W^a	W^e	z	z/z^a
Long-term contracts, $\alpha_z = 0$, $\kappa_M = 0$										
stdev	0.020	0.045	0.043	0.084	0.050	0.000	0.005	0.010	0.020	0.000
relstd	1.000	2.227	2.130	4.149	2.490	0.000	0.248	0.513	1.000	0.000
autocorr	0.887	0.937	0.760	0.887	0.887	0.000	0.981	0.887	0.887	0.000
corr(u)	-0.953	1.000	-0.814	-0.954	-0.954	0.000	-0.613	-0.954	-0.953	0.000
corr(z)	1.000	-0.953	0.948	0.999	0.999	0.000	0.499	1.000	1.000	0.000
elast(z)	1.000	-2.124	2.020	4.143	2.486	0.000	0.126	0.513	1.000	0.000
Long-term contracts, $\alpha_z = 0.302$, $\kappa_M = 0$										
stdev	0.020	0.084	0.084	0.158	0.095	0.004	0.009	0.018	0.027	0.008
relstd	1.000	4.152	4.144	7.820	4.694	0.167	0.449	0.897	1.338	0.374
autocorr	0.888	0.926	0.736	0.874	0.874	0.630	0.972	0.874	0.875	0.857
corr(u)	-0.953	1.000	-0.779	-0.943	-0.943	-0.014	-0.650	-0.943	-0.940	-0.814
corr(z)	1.000	-0.953	0.905	0.985	0.985	0.047	0.643	0.988	0.990	0.867
elast(z)	1.000	-3.957	3.747	7.702	4.623	0.019	0.292	0.886	1.325	0.325
Long-term contracts, $\alpha_z = 0.576$, $\kappa_M = 0$										
stdev	0.023	0.218	0.321	0.320	0.192	0.251	0.018	0.033	0.039	0.020
relstd	1.000	9.366	13.847	13.853	8.321	10.785	0.752	1.418	1.672	0.866
autocorr	0.904	0.609	0.654	0.844	0.844	0.052	0.954	0.845	0.845	0.815
corr(u)	-0.376	1.000	0.317	-0.361	-0.362	0.554	-0.289	-0.346	-0.336	-0.213
corr(z)	1.000	-0.376	0.654	0.901	0.901	0.176	0.844	0.909	0.912	0.599
elast(z)	1.000	-3.483	9.070	12.489	7.502	1.936	0.636	1.290	1.525	0.525
Long-term contracts, $\alpha_z = 0.576$, $\kappa_M = 6$										
stdev	0.021	0.182	0.225	0.361	0.217	0.073	0.018	0.038	0.039	0.021
relstd	1.000	8.835	10.887	17.560	10.561	3.371	0.872	1.852	1.880	1.014
autocorr	0.893	0.870	0.683	0.841	0.841	0.017	0.958	0.844	0.845	0.826
corr(u)	-0.863	1.000	-0.589	-0.864	-0.865	0.086	-0.622	-0.862	-0.850	-0.723
corr(z)	1.000	-0.863	0.774	0.913	0.913	0.143	0.787	0.928	0.933	0.741
elast(z)	1.000	-7.642	8.432	16.045	9.651	0.500	0.688	1.719	1.756	0.756
Long-term contracts, $\alpha_z = 0.576$, $\kappa_M = 12$										
stdev	0.020	0.188	0.211	0.371	0.223	0.005	0.019	0.040	0.039	0.021
relstd	1.000	9.385	10.596	18.546	11.162	0.248	0.923	2.024	1.938	1.055
autocorr	0.889	0.915	0.686	0.839	0.839	0.500	0.959	0.844	0.845	0.829
corr(u)	-0.925	1.000	-0.726	-0.919	-0.919	-0.030	-0.672	-0.924	-0.917	-0.807
corr(z)	1.000	-0.925	0.786	0.916	0.916	0.059	0.767	0.933	0.940	0.777
elast(z)	1.000	-8.678	8.324	16.987	10.223	0.028	0.710	1.890	1.823	0.823

Notes: cf. Table 2

Table 4: Elasticity of wages w.r.t. productivity and unemployment, US Data 1951-2003

	All		Male		Female	
	Stayers	Movers	Stayers	Movers	Stayers	Movers
Elasticity w.r.t. unemployment						
Aggregate data	-0.16					
Haefke, Sonntag, and van Rens (2006)	0.18	-1.30				
Hart (2003, Table 3)			-1.22	-2.01	-1.30	-1.70
Devereux and Hart (2005, Table 3)			-1.73	-2.92	-1.66	-2.49
Shin and Solon (2004, Tables 1-4)			$\approx$ -1.00			
Elasticity w.r.t. productivity						
Aggregate data	0.573					
Haefke, Sonntag, and van Rens (2006)	0.209	0.934				

Notes: external movers in the case of Devereux and Hart (2005)
Haefke, Sonntag, and van Rens (2006): Table 7, mean wages, 1984-2006,
corrected for education and demographics

Table 5: Elasticity of wages

Model	Elast. w.r.t. Z^{ave}			Elast. w.r.t. U		
	W^a	W^e	W^s	W^a	W^e	W^s
STC, $\alpha_z = 0$	0.944	0.944	0.944	-7.387	-7.387	-7.387
STC, $\alpha_z = 0.302$	1.461	1.631	1.460	-5.544	-6.154	-5.539
STC, $\alpha_z = 0.576$	2.912	3.431	2.910	-3.579	-4.199	-3.574
LTC, $\alpha_z = 0$, $\kappa_M = 0$	0.034	0.513	0.032	-0.588	-3.967	-0.553
LTC, $\alpha_z = 0.302$, $\kappa_M = 0$	0.091	0.946	0.087	-0.581	-3.444	-0.552
LTC, $\alpha_z = 0.576$, $\kappa_M = 0$	0.316	1.721	0.316	0.089	0.062	0.097
LTC, $\alpha_z = 0.576$, $\kappa_M = 6$	0.276	2.177	0.268	-0.243	-1.864	-0.227
LTC, $\alpha_z = 0.576$, $\kappa_M = 12$	0.269	2.344	0.260	-0.532	-2.969	-0.512

Notes: entries are regression coefficients.
Dependent variable: changes in the logs of W^a (average wage) or W^e (entry wage) or W^s (wage of job stayers)
Independent variable: changes in the log of z^a (average labor productivity) or the value of U (unemployment rate)
All variables are detrended.
STC: short-term contracts; LTC: long-term contracts;

A Proofs of Propositions 1 and 2

W.l.o.g., we can normalize $A_m = 1$. Define the total job surplus as $S(z, z_m) \equiv F(z, z_m) + V(z, z_m) - V^u(z_c)$. Adding (3a) and (3c) and subtracting (3b), we obtain an equation defining the surplus:

$$S(z, z_m) = Y(z, z_m) - b + \beta \, \mathrm{E}_{z'} \left[(1-\delta)S(z', z_m) - \alpha p^W(z) S(z', z')\right] \tag{19}$$

Take a first order approximation of (19) at the steady state:

$$\begin{aligned} S^* + S_c^* \tilde{z} + S_m^* \tilde{z}_m &= 1 + (1-\alpha_z)\tilde{z} + \alpha_z \tilde{z}_m - b \\ &+ \beta \, \mathrm{E}_{z'} \left[(1-\delta)(S^* + S_c^* \tilde{z}' + S_m^* \tilde{z}_m) - \alpha p_W^*(S^* + (S_c^* + S_m^*)\tilde{z}') - \alpha \frac{d\,p^W}{d\,z} S^* \tilde{z}\right] \end{aligned} \tag{20}$$

Here, the asterisk denotes steady state values, and the subscripts 'c' and 'm' denote partial derivatives with respect to current and match-time productivity, z and z_m, respectively.

If $\tilde{z}$ follows an AR(1) process, we can write $\mathrm{E}_{z'} \tilde{z}' = (1-\sigma)\tilde{z}$, and (20) simplifies to

$$\begin{aligned} S^* + S_c^* \tilde{z} + S_m^* \tilde{z}_m &= 1 + (1-\alpha_z)\tilde{z} + \alpha_z \tilde{z}_m - b \\ + \beta &\left[(1-\delta)(S^* + S_c^*(1-\sigma)\tilde{z} + S_m^* \tilde{z}_m) - \alpha p_W^*(S^* + (S_c^* + S_m^*)(1-\sigma)\tilde{z}) - \alpha \frac{d\,p^W}{d\,z} S^* \tilde{z}\right] \end{aligned} \tag{21}$$

Equ. (21) must hold for all values of $\tilde{z}$ and $\tilde{z}_m$; collecting terms we get

$$S^* = 1 - b + \beta(1 - \delta - \alpha p_W^*)S^* \tag{22a}$$

$$S_c^* = (1-\alpha_z) + \beta \left[(1-\delta)(1-\sigma)S_c^* - \alpha p_W^*(1-\sigma)(S_c^* + S_m^*) - \alpha \frac{d\,p^W}{d\,z} S^*\right] \tag{22b}$$

$$S_m^* = \alpha_z + \beta(1-\delta)S_m^* \tag{22c}$$

From (2) and (5), using $F(z, z) = (1-\alpha)S(z, z)$ at the time of the match, we obtain

$$p^W(z) = p^F(z)^{(\alpha-1)/\alpha} = \left(\kappa^{-1}(1-\alpha)S(z, z)\right)^{(1-\alpha)/\alpha} \tag{23}$$

Let us define, for any function x of z, the elasticity of x with respect to z at the steady state as $\eta_z^x \equiv \frac{d\,x}{d\,z}\big|_{z=1} \frac{1}{x(1)}$. Then we get from (23) that

$$\eta_z^{p^W} = \frac{1-\alpha}{\alpha} \eta_z^{S()} = \frac{(1-\alpha)}{\alpha} \frac{(S_c^* + S_m^*)}{S^*} \tag{24}$$

Using (24) in (22b), and adding up (22b) and (22c), we get

$$S_c^* + S_m^* = 1 + \beta \left[(1-\delta)(1-\sigma) - p_W^*(1-\alpha\sigma)\right](S_c^* + S_m^*) + \beta(1-\delta)\sigma S_m^* \tag{25}$$

Solving (22c) for S_m^* and plugging it into (25), we can solve for

$$S_c^* + S_m^* = \frac{1 + \beta(1-\delta)\sigma \frac{\alpha_z}{1-\beta(1-\delta)}}{1 - \beta\left[(1-\delta)(1-\sigma) - p_W^*(1-\alpha\sigma)\right]} \tag{26}$$

Solving (22a) for S^* and using (24) and using (26), we obtain Equ.(12) of Proposition 1. For the continuous time limit, replace β by $(1 - r\Delta)$, σ by $\sigma\Delta$, δ by $\delta\Delta$ and p^W by $p^W\Delta$, and take the limit $\Delta \to 0$. This gives Equ. (13).

For Proposition 2, we get from (4) that $F(z, z_m) = (1-\alpha)S(z, z_m)$ and $V(z, z_m) - V^u(z_c) = \alpha S(z, z_m)$. Then we can condense (3) to

$$\alpha S(z, z_m) = w(z, z_m) - b + \beta \, \mathrm{E}_{z'}\left[(1-\delta)\alpha S(z', z_m) - p^W(z)\alpha S(z', z')\right] \tag{27a}$$

$$(1-\alpha)S(z, z_m) = Y(z, z_m) - w(z, z_m) + \beta \, \mathrm{E}_{z'}\left[(1-\delta)(1-\alpha)S(z', z_m)\right] \tag{27b}$$

Subtract (27b), divided by $1-\alpha$, from (27a), divided by α, to obtain

$$w(z, z_m) = (1-\alpha)b + \alpha Y(z, z_m) + \alpha(1-\alpha)\beta p^W(z) \, \mathrm{E}_{z'} \, S(z', z') \tag{28}$$

Taking derivatives at the steady state we get

$$\begin{aligned} w_c &= \alpha(1-\alpha_z) + \alpha(1-\alpha)\beta\left[\frac{d\,p^W}{d\,z}S^* + p_W^*(1-\sigma)(S_c^* + S_m^*)\right] \\ &= \alpha(1-\alpha_z) + \beta\alpha(1-(1-\alpha)\sigma)\eta_z^{p^W} p_W^* S^* \qquad (29) \\ w_m &= \alpha\alpha_z \qquad (30) \end{aligned}$$

Solving (22a) for S^* and inserting into (29) we get Equ. (15) in Proposition 2. For the continuous time limit (16), take the same steps as above.