Macroeconomics III: Task list 2

To be handed in:

All answers should be SHORT AND PRECISE! Good luck!

1) The household maximizes

$$\max \mathrm{E} \sum_{t=0}^{\infty} \beta^t \left[\log(c_t - hc_{t-1}) + \eta \log(1 - L_t)\right] \tag{1}$$

where h is a habit parameter, subject to the law of motion for end-of-period assets A_t:

$$A_t = (1 + r_t)A_{t-1} + w_t L_t - C_t \tag{2}$$

r_t is the stochastic return to the asset.

Derive the household labor supply equation (trade-off consumption vs. leisure) and interpret it.

Solution: Lagrangian:

$$\mathcal{L} = \mathrm{E} \sum_{t=0}^{\infty} \beta^t \left[\log(c_t - hc_{t-1}) + \eta \log(1 - L_t) + \lambda_t \left(-A_t + (1 + r_t)A_{t-1} + w_t L_t - c_t\right)\right] \tag{3}$$

Then

$$\frac{\partial \mathcal{L}}{\partial c_\tau} = \frac{1}{c_\tau - hc_{\tau-1}} - h\,\mathrm{E}_\tau \frac{1}{c_{\tau+1} - hc_\tau} - \lambda_\tau = 0 \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial L_\tau} = -\eta \frac{1}{1 - L_\tau} + w_t \lambda_\tau = 0 \tag{5}$$

Then,

$$w = \eta \frac{\lambda_t^{-1}}{1 - L_t} = \eta \frac{\frac{1}{c_\tau - hc_{\tau-1}} - h\,\mathrm{E}_\tau \frac{1}{c_{\tau+1} - hc_\tau}}{1 - L_t} \tag{6}$$

Here, λ_t is the marginal effect of an increase in c_t, which in the model without habit is the marginal utility of consumption.

2) **RBC Model with government expenditure shocks**

The model is an RBC model with exogenous shocks to government spending, see Equs. (**??**) and (**??**). It is financed by labor taxes τ_t such that the budget is balanced in every period, cf. (**??**). Taxes are distortive, cf. (**??**). The aggregate resource constraint is (**??**). Government expenditures affect household utility, cf. (**??**).

Figure 1 shows the responses to a government expenditure shock. Figure 2 shows the impulse responses to a technology shock. Notice that all the impulse responses are in percentage deviations from steady state. Only the interest rate is in *percentage point* deviations.

a) Compute the steady state of the RBC model. Choose the parameters A and $\bar{L}$ such that $Y = 1$ and $L = 1$ in steady state.

Solution: From (**??**) we get in steady state $z = 0$. From the Euler equation we get that $r = \beta^{-1} - 1$. Short route: from the properties of the Cobb-Douglas function we get $(r+\delta)K = \alpha Y = \alpha$, therefore $K = \frac{\alpha}{r+\delta}$. From the production we get then

$$1 = Y = A\left(\frac{\alpha}{r+\delta}\right)^{\alpha} \tag{7}$$

Therefore

$$A = \left(\frac{r+\delta}{\alpha}\right)^{\alpha} \tag{8}$$

In steady state, $I = \delta K = \frac{\alpha\delta}{r+\delta}$. Then $c = Y - G - I = 1 - 0.2 - \frac{\alpha\delta}{r+\delta}$. Wage is $(1-\alpha)Y/L = (1-\alpha)$. Tax rate is then $\tau = g/(wL) = 0.2/(1-\alpha)$. Labor supply condition with utility function (**??**) then says

$$(1-\alpha)(1-0.2/(1-\alpha)) = \eta\frac{c}{\bar{L}-1} = \eta\frac{0.8-\frac{\alpha\delta}{r+\delta}}{\bar{L}-1} \tag{9}$$

and therefore

$$\bar{L} = 1 + \eta\frac{0.8-\frac{\alpha\delta}{r+\delta}}{0.8-\alpha} \tag{10}$$

b) Explain why labor input declines after a positive shock to government expenditures (Figure 1).

Explain why investment declines strongly on impact.

Solution: Income effect (households are poorer because government needs resources and spends them in a way that does not affect the marginal utility of private consumption) would lead to an increase in labor (less leisure). Higher taxes create substitution effect towards less labor, which dominates here. Households choose to work less, finance their consumption by a reduction in saving, and compensate this by working more later, when tax has decreased.

c) How would the response of labor and investment change if government expenditures were financed by lump sum taxes?

Solution: Only substitution effect: labor goes up, consumption down, investment goes also down, but by less.

d) Explain why labor input increases when a positive productivity shock hits (Figure 2). Explain the different effects at work.

Solution: Substitution effect: higher wage leads to higher labor supply. Income effect: higher lifetime wealth leads to more demand for leisure, smaller labor supply. Usually, substitution effect is bigger, since productivity shock is transitory.

With this version of government expenditures, higher productivity and output means that the same expenditures (G depends on steady state output, not actual output) can be financed by lower tax rate, so that the positive substitution effect on labor input increases even more.

e) Explain why the response of labor fades out quickly, although wages peak at around period 20.

Solution: The initial increase in L comes largely from the reduction of the labor tax rate; since the tax rate goes back to normal quickly, so does labor supply. The above steady state interest rate also contributes somewhat, since it means, ceteris paribus, that demand for leasure should increase in the first 20 periods.

f) Explain the reaction of investment on impact.

Solution: Consumption smoothing means that households want to save most of the additional resources; the higher productivity of capital makes firms to hire more capital, so that more is invested.

g) Explain the relationship between the responses of consumption and of the real interest rate.

Solution: From the HH Euler equation it follows that the change in consumption is positive while the interest rate is above the steady state level; this turns around after approximately 20 periods.

3) **Merz Model**

The wage equation in the Merz model is

$$w_t = \lambda \left[F_L(z_t, k_t, N_t) + a \frac{V_t}{1 - N_t} \right] + (1 - \lambda) \left[\frac{U_L(c_t, N_t)}{U_c(c_t, N_t)} - c(S_t) \right] \tag{11}$$

where λ is the bargaining power of workers, a is the vacancy cost, $c(S)$ are search costs,

Interpret each term in this equation.

Solution: If the bargaining power of the worker, λ, goes to 1, wages are deriven up to the marginal current value of a job to the employer:

- the marginal productivity of labor
- what they save in tersm of vacancy posting costs

If the bargaining power of the worker goes to 0, wages are driven down to the marginal current value of a job to the worker:

- the marginal disutility of labor, expressed in units of the consumption good (divide by U_c)
- the value of not having to search

Figure 1: RBC model, IR to government expenditure shock

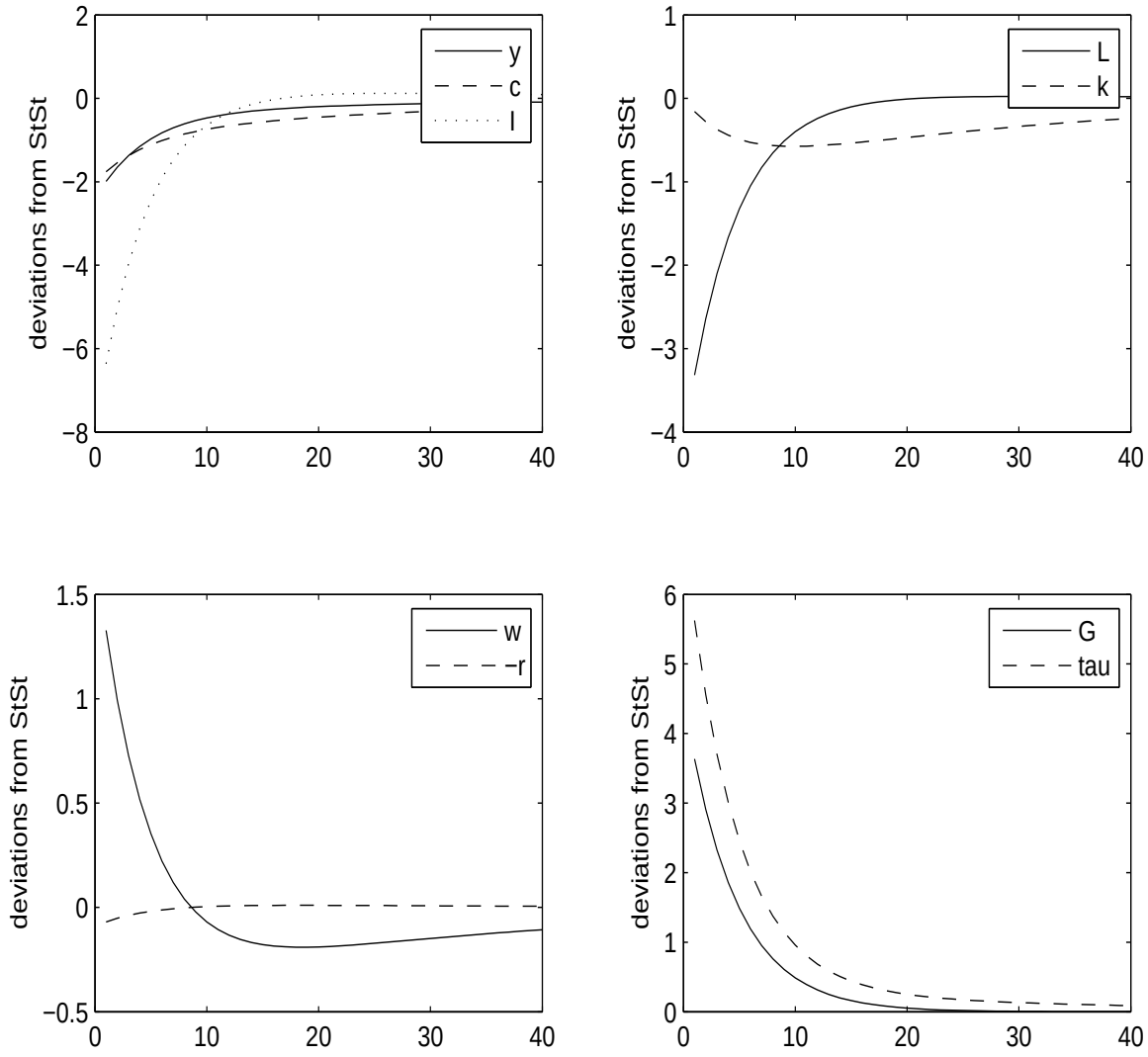

Figure 2: RBC model, IR to technology shock

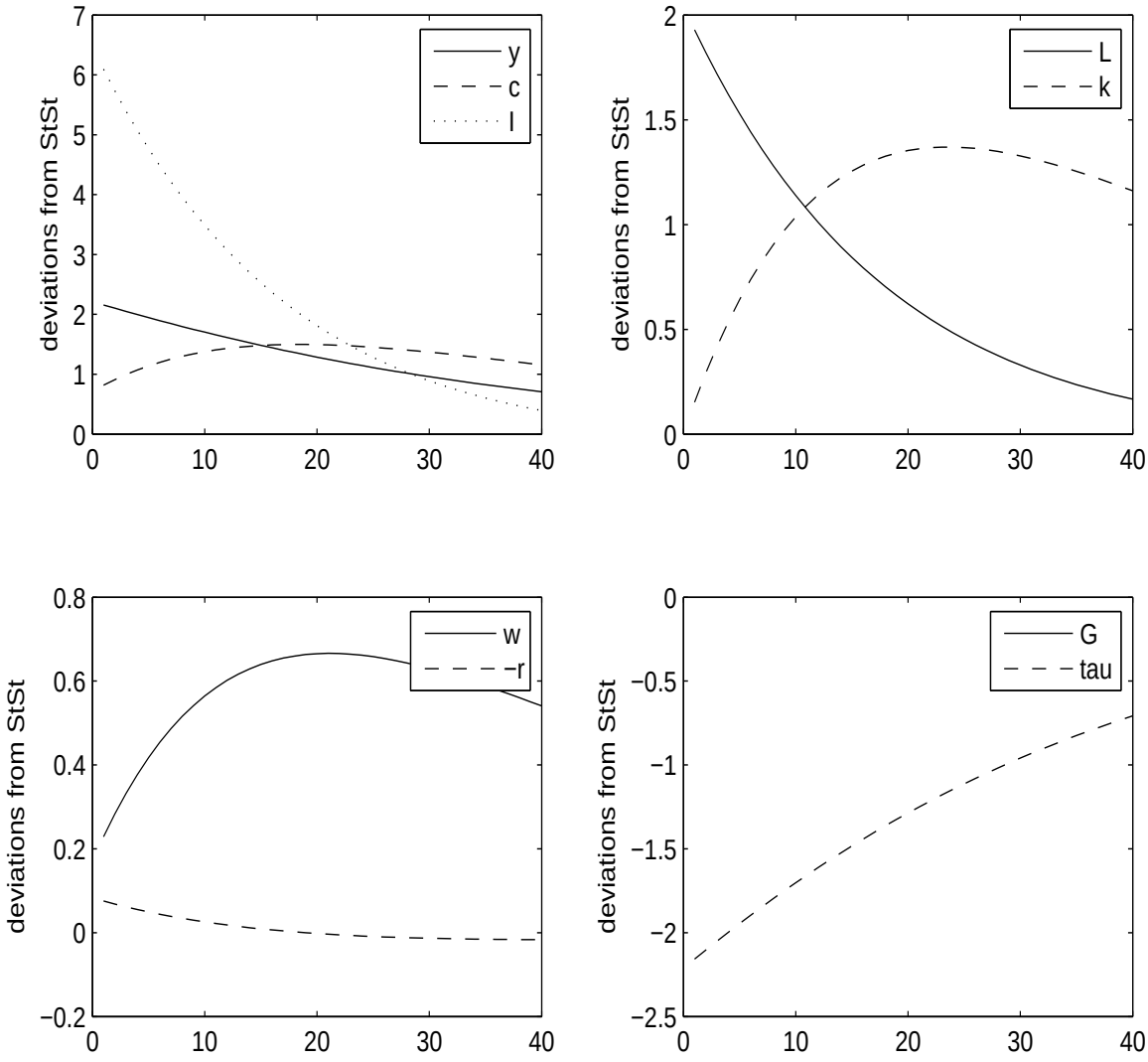

Figure 3: IR to tech shock, Coles/Moghaddasi model

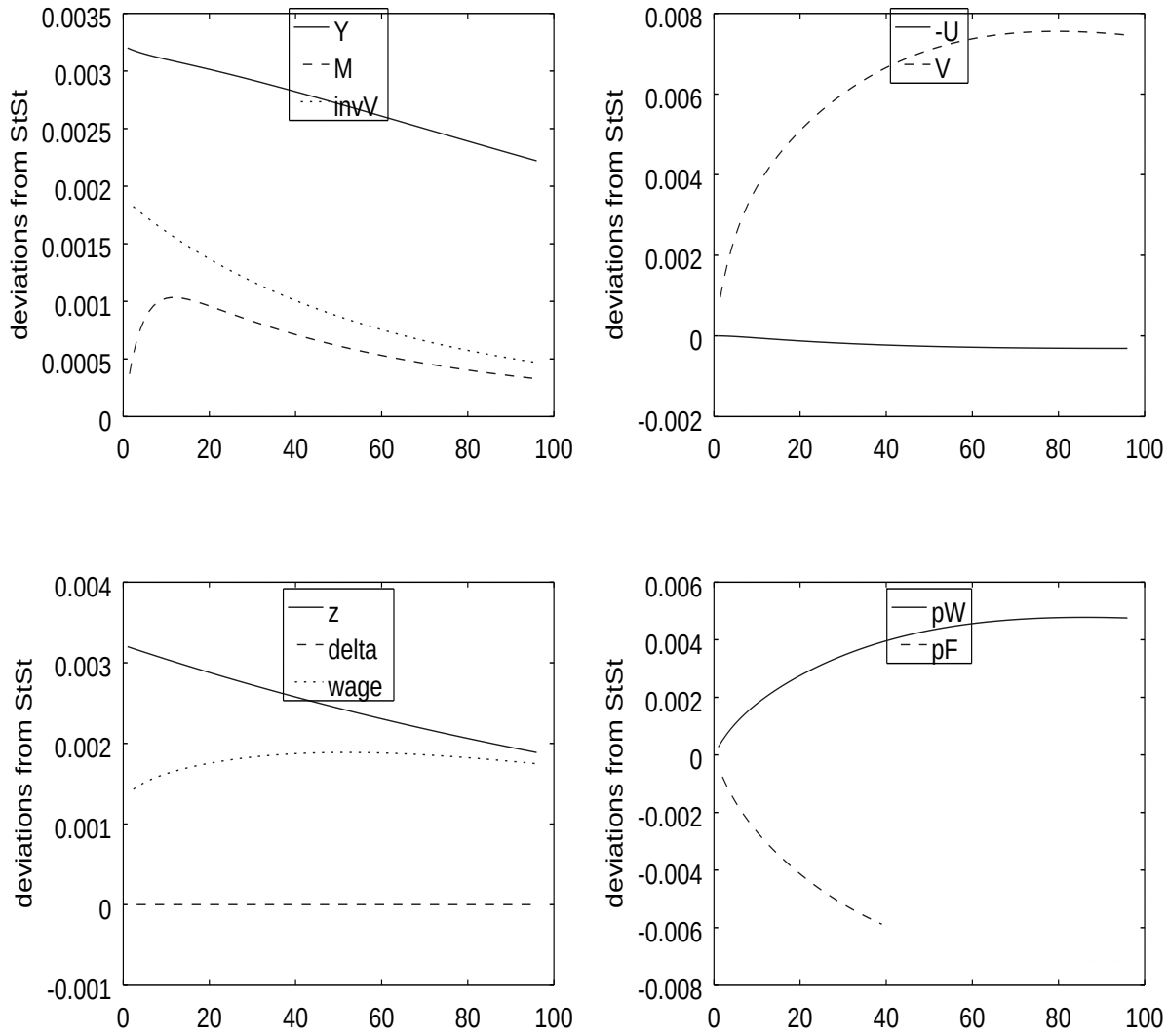

Figure 4: IR to separation shock, Coles/Moghaddasi model