Macroeconomic Theory II: Exercise list 4

To be handed in by February 12, 2020

1) Take the RBC model of the class handout.

a) Derive the first order conditions of the planner solution. **Solution:**
The Lagrangian of the planner is

$$\mathcal{L} = \mathrm{E}\sum_{t=0}^{\infty}\beta^t\left[U(c_t,L_t)+\lambda_t\left(-K_t+(1-\delta)K_{t-1}+F(K_{t-1},L_t,z_t)-c_t\right)\right] \tag{1}$$

FOCs are:

$$\beta^t\frac{\partial\mathcal{L}}{\partial c_\tau} = U_c(c_t,L_t)-\lambda_t = 0$$
$$\beta^t\frac{\partial\mathcal{L}}{\partial L_\tau} = U_L(c_t,L_t)+\lambda_t F_L(K_{t-1},L_t,z_t) = 0$$
$$\beta^t\frac{\partial\mathcal{L}}{\partial K_\tau} = -\lambda_t+\mathrm{E}_t\,\lambda_{t+1}\left((1-\delta)+F_K(K_t,L_{t+1},z_{t+1})\right) = 0 \tag{2}$$

which gives

$$\lambda_t = U_c(c_t,L_t)$$
$$U_L(c_t,L_t) = -U_c(c_t,L_t)F_L(K_{t-1},L_t,z_t) \tag{3}$$
$$U_c(c_t,L_t) = \mathrm{E}\left[U_c(c_{t+1},L_{t+1})\left((1-\delta)+F_K(K_t,L_{t+1},z_{t+1})\right)\right] \tag{4}$$

Notice

* It is good to define K_t as capital at the end of the period, and therefore write $F(K_{t-1},L_t,z_t)$ rather than $F(K_t,L_t,z_t)$, because K_t is already known at time t, in this way there is no mistake about where we need an expectation.
* The planner's problem has no prices, only allocations.

b) Compare them to the equations that characterize the decentralied solution. Show that a solution to one set of equations is also a solution to the other set of equations.
Solution:
We compare this to the decentralized equations

$$U_c(c_t,L_t) = \beta(1+r_{t+1})U_c(c_{t+1},L_{t+1}) \tag{5}$$
$$F_k(z_t,k_{t-1},L_t) = (r_t+\delta) \tag{6}$$
$$w_t = F_L(z_t,k_{t-1},L_t) \tag{7}$$
$$0 = w_tU_c(c_t,L_t)-U_L(c_t,L_t) \tag{8}$$

Solving for t_t in (6) and plugging it into (5), (5) becomes identical to (4). Plugging the wage from (7) into (8), (8) becomes identical to (3). This shows that any allocation that satisfies the decentralized equations also satisfies the solution to the planner's problem.

In the opposite direction: any allocation that satisfies the planner FOCs also satisfy the decentralized equations by introducing shadow prices r_t and w_t given by (6) and (7).

One can add that the transversality conditions for the household problem and the planner problem are also equivalent given (6).

2) **Demand function with Dixit-Stiglitz aggregator** There are N firms, each produing a differentiated good. Firm j posts price P_j.

The representative household has utility function $U(C, L)$ where C is defined as

$$C = N^{\frac{1}{1-\eta}} \left(\sum_{j=1}^{N} C(j)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{9}$$

and $C(j)$ is the quantity of good j.

The household budget constraint is

$$\sum_{j=1}^{N} P_j C(j) = Y \tag{10}$$

where Y is some given expenditure level. Derive the FOC for the household and express $C(j)$ as a function of $P(j)$, Y and the price level P, which is defined as

$$P \equiv \left(\frac{1}{N} \sum_{j=1}^{N} P_j^{1-\eta} \right)^{\frac{1}{1-\eta}} \tag{11}$$

Solution:
Lagrangian:

$$\mathcal{L} = N^{\frac{1}{1-\eta}} \left(\sum_{j=1}^{N} C_{ji}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} + \lambda \left(Y - \sum_{j=1}^{N} P_j C(j) \right) \tag{12}$$

First order condition:

$$\frac{\partial \mathcal{L}}{\partial C(j)} = N^{\frac{1}{1-\eta}} \Phi^{\frac{-1}{1-\eta}} C_{ji}^{\frac{-1}{\eta}} - \lambda P_j = 0 \tag{13}$$

where

$$\Phi \equiv \left(\sum_{j=1}^{N} C(j)^{\frac{\eta-1}{\eta}} \right) \tag{14}$$

This gives

$$C_{ji} = (N/\Phi)^{\frac{\eta}{1-\eta}} (\lambda P_j)^{-\eta} \tag{15}$$

Integrating over goods and using the budget constraint gives

$$\sum_j P_j C_{ji} = (N/\Phi)^{\frac{\eta}{1-\eta}} \lambda^{-\eta} \sum_j P_j^{1-\eta} = Y \tag{16}$$

Using (16) and (11) in (15) gives

$$C(j) = Y \frac{P_j^{-\eta}}{\sum_i P_i^{1-\eta}} = \frac{Y P_j^{-\eta}}{P^{1-\eta} N} \tag{17}$$

Then

$$P \cdot C = PN^{\frac{1}{1-\eta}} \left(\sum_{j=1}^{N} C_{ji}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{18}$$

$$= PN^{\frac{1}{1-\eta}} \left(\sum_{j=1}^{N} \left(Y P_j^{-\eta} P^{\eta-1} N^{-1} \right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{19}$$

$$= PN^{\frac{1}{1-\eta}} \left(\sum_{j=1}^{N} P_j^{1-\eta} \right)^{\frac{\eta}{\eta-1}} Y P^{\eta-1} N^{-1} \tag{20}$$

$$= PN^{\frac{1}{1-\eta}} \left(N P^{1-\eta} \right)^{\frac{\eta}{\eta-1}} Y P^{\eta-1} N^{-1} = Y \tag{21}$$

3) Write a Dynare file that solves the model in (Merz 1995). Use the parameters values given in the paper.

a) Before you start programming, write down the list of all variables and all equations in the model, make sure that the number of variables equals the number of equations, and that all the variables in your equations are contained in the list of variables!

b) To compute the steady state, write a matlab file that takes as input a guess for N^* and a guess for S^*, computes the steady state values of all other variables (given the guess of N^* and S^*), and returns a vector for two residuals that is zero if the guesses are right.

Use the Matlab function "fsolve" to find the equilibrium N^* and S^*.

c) Write the Dynare file with all the model equation. Give the steady state that you have computed as starting values for the model variables. Solve the model with first order approximation.

Check whether the steady state that Dynare find corresponds to your own steady state calculation.

Compare the impule responses produced by Dynare with the ones in the paper.

References

Merz, M. (1995). Search in the labor market and the real business cycle. *Journal of Monetary Economics 36*, 266–300.