Macroeconomic Theory II: Exercise list 5

To be handed in by 15.3.2020

1) **Monetary policy in the NK model**

Consider the setup of Clarida/Gali/Gertler:

$$x_t = \mathrm{E}_t\, x_{t+1} - \phi(i_t - \mathrm{E}_t\, \pi_{t+1}) + g_t \tag{1}$$
$$\pi_t = \beta\, \mathrm{E}_t\, \pi_{t+1} + \lambda x_t + u_t \tag{2}$$
$$g_t = \mu g_{t-1} + \epsilon_{g,t} \tag{3}$$
$$u_t = \rho u_{t-1} + \epsilon_{u,t} \tag{4}$$

The objective of the government is to maximize

$$\frac{1}{2}\, \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\alpha x_t^2 + \pi_t^2\right] \tag{5}$$

a) First assume the case where the central bank cannot commit at all to future policies, as in CGG. Write down the optimization problem of the central bank (don't forget the constraints!) in any period t. Use the same kind of simplification as Clarida/Gali/Gertler in their paper.

b) Derive the FOCs of the central bank problem for t and $t+1$.

c) Now assume that at time 0 the central bank can credibly set the policy for periods 0 and 1 (of course, in $t = 1$ the policy is conditional on state variables in $t + 1$). From $t = 2$ onwards, a new policy will be set without recognizing any previous commitment.

   Write the optimization problem of the central bank in period 0, using the same tricks to simplify the problem. Derive the FOCs of the central bank problem.

d) Explain how the ability to commit in 0 for period 1 affects the solution.

e) Assume a policy rule of the form $x_t = -\omega \pi_t$ for any constant ω.

   Derive the interest rate rule that implements this policy rule.

   Derive the variances of inflation and the output gap that result from this rule.