Macroeconomic Theory II, CEU: Midterm exam

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All answers should be SHORT AND PRECISE! If something is unclear, write down what you think is unclear, make an assumption and proceed.

Good luck!

1) **Impulse responses in the RBC model**

As a reference, the formulas of the model are given in Appendix A. The impulse responses are given in Figure 1.

a) On impact of a positive technology shock, labor input increases. Why is this the case? Explain the different effects at work.

Does the positive response or labor get bigger or smaller if we reduce the persistence of the techology shock ρ? **Solution:**
The increase in TFP leads to an increase in the MPL and therefore of the wage. This implies a substitution effect that tends to increase labor supply. At the same time, the increased value of the time endowment of the HH means that HHS feel richer (income or wealth effect), and since leisure is a normal good, this tends to decrease labor supply. That labor input goes up on impact means that the substitution effect dominates.

If TFP is less persistent, there is a smaller wealth effect, since the relevant wealth concept is expected discounted wealth from now to the infinite future. Since the wealth effect is diminished, and labor input increases even more.

b) After the positive technology shock consumption is still increasing (after the initial upward jump) in the first part of the response function. Is this because the interest rate is higher than in steady state, or because capital is increasing over this part of the sample, due to higher investment (wealth effect)? Which of these explanations is correct (could be none or both)? **Solution:**
If there is no shock, the change in consumption is determined only through the real interest rate. Consumption is increasing as long as the interest rate is higher than in steady state. The fact that capital is increasing plays no role here.

c) The real wage is also increasing over the first part of the reponse function, although the technology parameter is gradually returning to the steady state level. How can this be explained? **Solution:**
The real wage is equal to the marginal productivity of labor. Although TFP returns to its steady state level, the increase in the stock of capital, and the continuous reduction of labor input explain why MPL goes up in the first part of the sample.

2) If we introduce government expenditures into the RBC model, and assume they are financed by labor taxes, how does the labor supply equation in the model change?

   Explain why the decentralized equilibrium is not Pareto efficient in this case (compare to first order conditions of the planner). **Solution:**
   We dnote by w the before-tax real wage. The HH FOC implies that the after tax real wage $w(1-\tau)$, rather than w, equals the ratio of MUL to MUC.

   The firms pay the before tax wage. Their FOC then says that the before tax wage equals the MPL. This implies that $MPL = w < w(1-\tau) = MUL/MUC$. In contrast, the FOC of the planner requires This implies that $MPL = MUL/MUC$.

3) Consider the RBC model with utility function

$$U(c, L) = \log c + \eta \log(1 - L) \tag{1}$$

   and production function

$$Y_t = Ae^{z_t} K_t^{\alpha} L_t^{1-\alpha} \tag{2}$$

   (otherwise same model as in Appendix A). Compute the deterministic steady state of the RBC model and calibrate the parameters A and η such that in steady state $K = 1$ and $L = 1/3$. **Solution:**
   We get $z = 0$ from the AR process, and $r^* = 1/\beta - 1$ from the HH Euler equation. The FOC for capital input then implies $r^K = r^* + \delta = A\alpha K_t^{*\alpha} L_t^{*(1-\alpha)} = A\alpha(1/3)^{1-\alpha}$, therefore we get the calibration $A = \frac{1/\beta - 1 + \delta}{\alpha}$, and from the production function we get $Y^* = A$. From the FOC for labor input we obtain $w^* = A(1-\alpha)$. The dynamic equation for capital implies $I^* = \delta K^* = \delta$. The $c^* = Y^* - I^* = A - \delta$. Finally we use $w^* = MUL/MUC = \eta \frac{c^*}{1-L^*}$ to calibrate $\eta = \frac{2}{3} \frac{w^*}{c^*}$

4) **Impulse responses in the New Keynesian model**

The model is from Chapter 3 of the Gali book. It is a model without capital or investment, so $Y = C$. Monetary policy is given by the Taylor rule

$$\log(R_t/R^*) = \gamma_\pi \log(\Pi_t/\Pi^*) + mshock_t \tag{3}$$

where the monetary policy disturbance $mshock_t$ is autocorrelated:

$$mshock_t = \rho_m mshock_{t-1} + \epsilon_{m,t} \tag{4}$$

Parameters are $\gamma_\pi = 1.5$ and $\rho_m = 0.5$.

The upper panels in Figure 2 show the impulse responses to a 1% technology shock. The lower panels show impulse responses to a monetary policy shock (1 percentage point **decrease** in the nominal rate).

Notice that responses for interest and inflation are *percentage point* deviations from steady state, while the other variables are *percentage* deviations from steady state,

a) For both types of shock, what explains the consumption response? **Solution:**
As always, the shape of the consumption path is related to the expected real interest rate. Consumption returns to the steady state, and if the interest rate is above the steady state, consumption must decrease over time. In order to decrease over time and return to steady state, it has to start above the steady state level.

Notice that the logic of income or wealth effect is somewhat different in the NK model compared to the RBC model. The wage does not exactly follow MPL, because of endogenous changes in the markup. The extent to which higher productivity translates into higher income depends on the quantity produced, which is driven by demand. Notice, however, that the income effect for households does not just depend on the wage, but also on profit income.

The increase in the real interest rate is a consequence of monetary policy. In the case of the technology, inflation decreases, because the increase in productivity lowers the real marginal cost of firms. They therefore tend to decrease prices, which lowers inflation below the steady state. Using the Taylor rule, the central bank reduces the nominal rate by more than inflation, so that the real rate decreases.

In case of the monetary policy shock, the nominal rate decreases because of the shock.

b) Labor input decreases on impact of the technology shock. Explain why this happens. **Solution:**
Since price is higher than marginal cost, firms produce as much as is demanded. In this model, demand is consumption demand, and consumption is explained by the path of the real interest rate, as explained above. It can then happen (not always the case), that the increase in demand is lower than the increase in production that results from higher productivity, at a given level of labor input. In this case, the new level of demand can be produced with a reduction in labor input.

c) Why does inflation respond negatively to the technology shock? **Solution:**
This was already explained above. In the case of the technology shock, inflation decreases, because the increase in productivity lowers the real marginal cost of firms. They therefore tend to decrease prices, which lowers inflation below the steady state.

d) Why does inflation respond positively to the monetary shock? **Solution:**
The negative shock to the nominal rate is a reduction in the real rate (for a given level of inflation). This has an expansionary effect on demand (consumption), which leads firms to produce more, which requires more labor, so that firms have to pay a higher wage, which increases marginal costs, which induces firms to charge higher prices, which leads to inflation.

5) **Mortensen/Pissarides (MP) Model**

The formulas of the model are given in Appendix B.

Figure (3) shows the impulse responses to a technoloyg shock in the MP model.

a) Explain why vacancies V go up and unemployment U goes down on impact. **Solution:**
Increase in productivity increases the surplus from a match (difference value match to value unemployed); the increased surplus is split between firm and worker, this means that the firm is willing to post a vacancy even if pF is lower (lower probability compensated by higher profit); from the mechanics of the matching function, lower pF (higher tightness) implies higher pW.

b) The reduction in unemployment is small compared to real world fluctuations. (Notice that the change in unemployment U is shownn as a percentage, not a *percentage point* change!) What explains this?

What could one change to make unemployment more volatile? **Solution:**
The equilibrium wage goes up by almost as much as productivity, so that very little of the extra production is left as profit for the firm, so that there is only a small increase in the incentive to post vacancies. One reason why the wage changes so much is because any increase p^W improves the bargaining position of workers.

Unemployment fluctuates more if

(a) wages are rigid, for whatever reason

(b) the "unemployment benefit" b is higher.

The percentage change in pF comes from the percentage (proportional) change in surplus. If $b - 0.4$, the average surplus is already high, so an increase in productivity causes a small percentage change in the surplus. Assuming, for example, $b = 0.95$, the average surplus is small, so that a one percent change in labor productivity impliesa large proportional change in the instantaneous surplus $\exp(z_t) - b$.

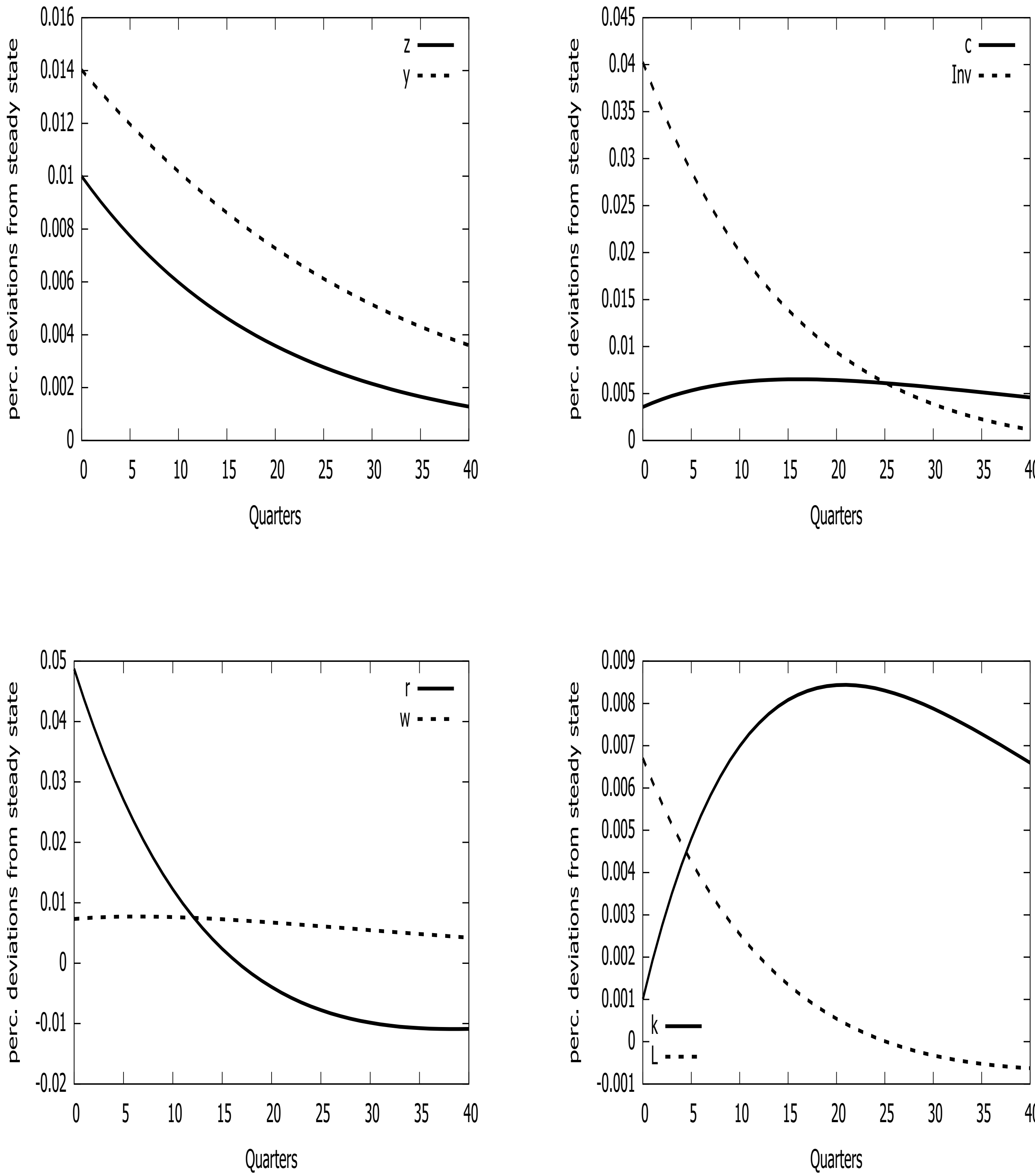

Figure 1: Impulse responses in the RBC model

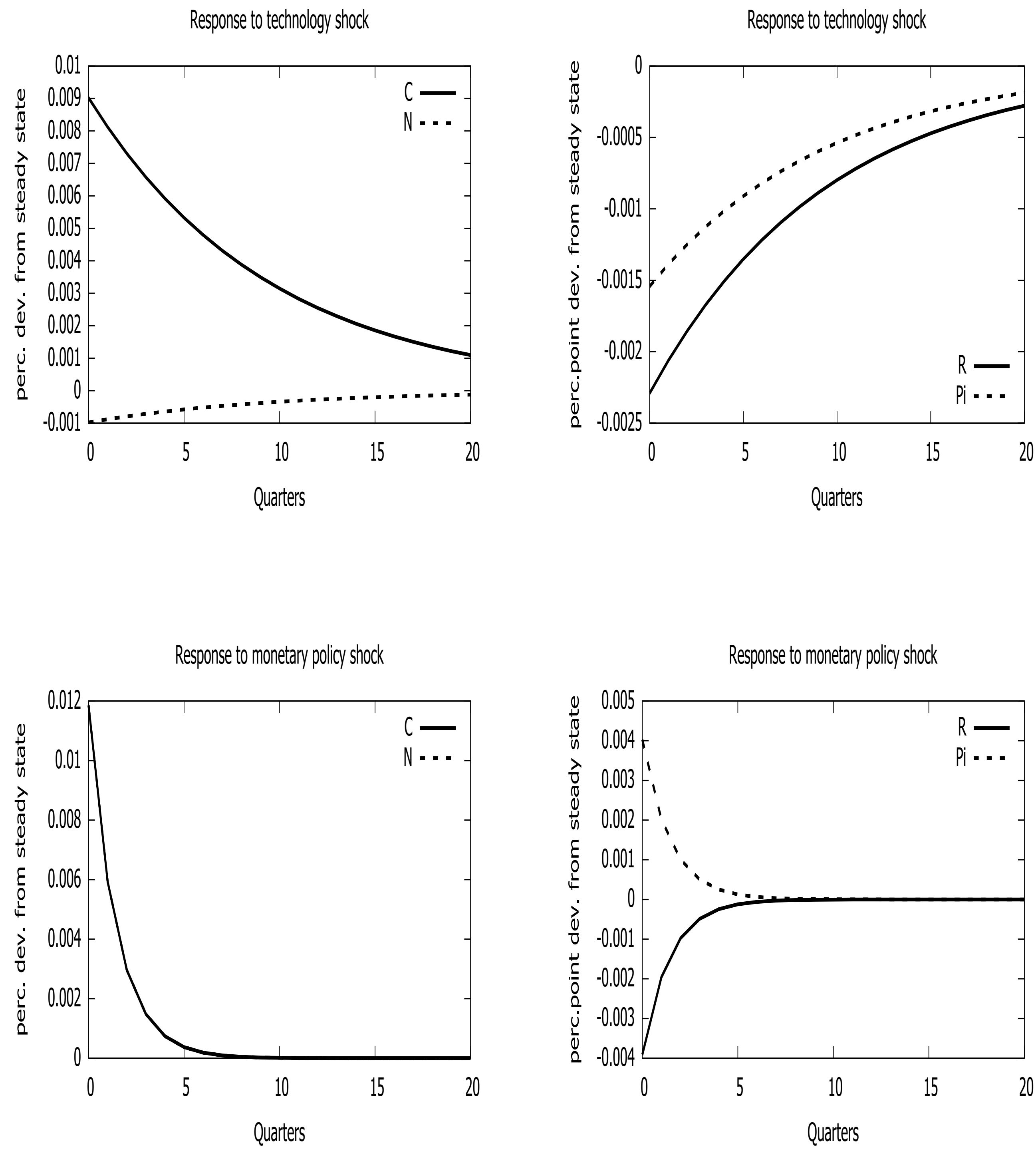

Figure 2: Impulse responses in the New Keynesian model

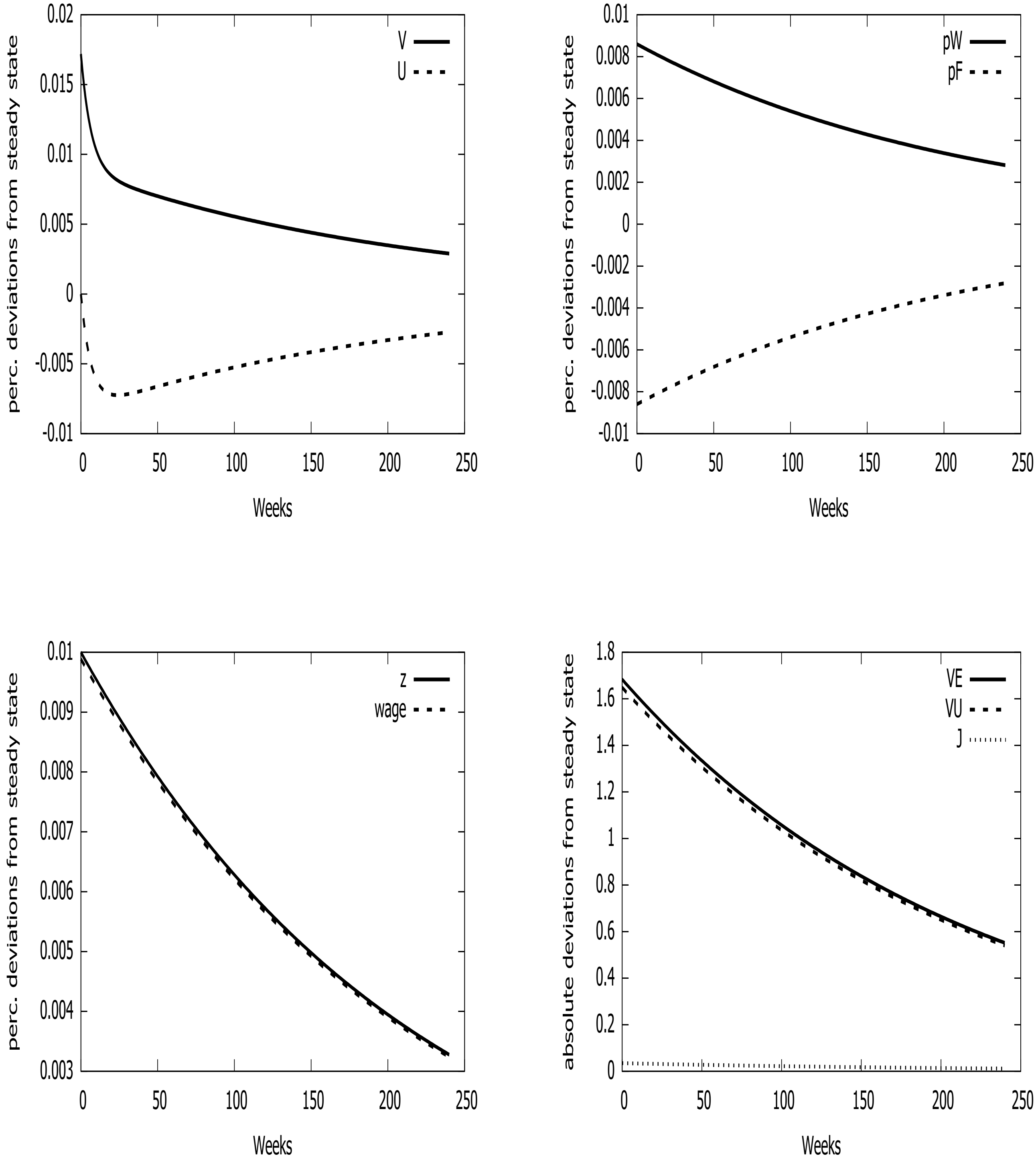

Figure 3: Impulse responses in the MP model

A The RBC Model

discount factor: $\beta = 0.99$; depreciation rate for capital: $\delta = 0.025$; output share of capital: $\alpha = 0.4$; weight of leisure in utility: $\eta = 1.5$; autocorrelation of technology shock: $\rho = 0.95$; target for capital-output ratio: $K_Y = 12$; Marginal utility of consumption:

$$U_c(c, l) = 1/c \tag{5}$$

Marginal utility of leisure:

$$U_L(c, l) = \eta/(1 - l) \tag{6}$$

production function:

$$F(z, k, l) = zk^{\alpha} l^{(1-\alpha)} \tag{7}$$

Marginal productivity of capital:

$$F_k(z, k, l) = \alpha z(l/k)^{1-\alpha} \tag{8}$$

Marginal productivity of labor:

$$F_L(z, k, l) = (1 - \alpha)z(k/l)^{\alpha} \tag{9}$$

Exogenous equation for productivity:

$$\log(z(0)) = \rho \log(z(-1)) + \epsilon_t \tag{10}$$

Household Euler equation:

$$U_c(c_t, L_t) = \beta(1 + r_{t+1})U_c(c_{t+1}, L_{t+1}) \tag{11}$$

Optimal capital input: gross interest rate = marginal productivity of capital:

$$F_k(z(0), k_{t-1}, L_t) = (r_t + \delta) \tag{12}$$

Production function

$$y_t = F(z(0), k_{t-1}, L_t) \tag{13}$$

Law of motion for capital:

$$I_t = k_t - (1 - \delta)k_{t-1} \tag{14}$$

Aggregate resource constraint:

$$c_t = y_t - I_t \tag{15}$$

Optimal labor input: wage = marginal productivity of labor:

$$w_t = F_L(z(0), k_{t-1}, L_t) \tag{16}$$

Residual: labor supply equation:

$$0 = w_t U_c(c_t, L_t) - U_L(c_t, L_t) \tag{17}$$

B The MP Model

Matching function:

$$\mathcal{M}(V,U) \equiv \mu V^{1-\lambda} U^{\lambda} \tag{18}$$

Model equations:

$$z_t = \rho z_{t-1} + \epsilon_t \tag{19}$$

$$N_t = (1-\sigma) N_{t-1} + M_{t-1} \tag{20}$$

$$U_t = 1 - N_t \tag{21}$$

$$M_t = \mathcal{M}(V_t, U_t) \tag{22}$$

$$\theta_t = \frac{V_t}{U_t} \tag{23}$$

$$p_t^W = \frac{M_t}{U_t} \tag{24}$$

$$p_t^F = \frac{M_t}{V_t} \tag{25}$$

$$V_t^E = w_t + \beta \, \mathrm{E}_t \left[(1-\sigma) V_{t+1}^E + \sigma V_{t+1}^U \right] \tag{26}$$

$$V_t^U = b + \beta \, \mathrm{E}_t \left[p_t^W V_{t+1}^E + (1 - p_t^W) V_{t+1}^U \right] \tag{27}$$

$$J_t = \exp(z_t) - w_t + \mathrm{E}_t \left[\beta((1-\sigma)) J_{t+1} \right] \tag{28}$$

$$\alpha J_t = (1-\alpha)((V_t^E - V_t^U)) \tag{29}$$

$$Y_t = N_t \exp(z_t) \tag{30}$$

$$\kappa = \beta p_t^F \, \mathrm{E}_t \left[J_{t+1} \right] \tag{31}$$

Calibrations:

$$N_t = 0.94 \tag{32}$$

$$p_t^F = 0.1 \tag{33}$$