3 Notes on Imperfect Competition

3.1 A simple model of imperfect competition

There are N (many) households and firms in the economy.

Each firm produces a different good and sets its price.

Each household has a share in all the existing firms.

The households

Utility function of household i :

$$U_i = C_i - \frac{1}{\gamma} L_i^{\gamma}, \qquad \gamma > 1 \tag{1}$$

- The household sells its labor in the labor market (does not work in own firm).
- $\gamma > 1$: increaing marginal disutility of labor
- The consumption index C_i is a function of all the goods that the household consumes:

$$C_i = N^{\frac{1}{1-\eta}} \left(\sum_{j=1}^{N} C_{ji}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \qquad \eta > 1 \tag{2}$$

  where C_{ji} is the quantity of Q_j consumed by household i.
- Budget constraint:

$$\sum_{j=1}^{N} P_j C_{ji} = \pi_i + W L_i \tag{3}$$

where π_i is the profit income of household i.

The household maximizes (1) under the restriction (3). With the optimal quantities coming out of the household problem, we get

$$PC_i = \sum_{j=1}^{N} P_j C_{ji} \tag{4}$$

where P is the price index:

$$P \equiv \left(\frac{1}{N} \sum_{j=1}^{N} P^{1-\eta} \right)^{\frac{1}{1-\eta}} \tag{5}$$

Using (4), the budget constraint (3) simplifies to

$$PC_i = \pi_i + W L_i \tag{6}$$

Maximizing (1) with respect to L s.t. (6) we get the first order condition

$$\frac{\partial U_i}{\partial L_i} = \frac{W}{P} - L_i^{\gamma-1} = 0 \tag{7}$$

which implies

$$L_i = \left(\frac{W}{P}\right)^{\frac{1}{\gamma-1}} \tag{8}$$

Since $\gamma > 1$, the supply of labor is increasing with respect to the real wage.

The firms

- Production function of firm j:
$$Q_j = L_j \tag{9}$$
- $Q_j =:$ production
  $L_j =:$ input of labor
  The firm buys labor in the labor market, which is competitive.
- Demand function of the household:
$$C_{ij} = \frac{1}{N} Y_i \left(\frac{P_j}{P}\right)^{-\eta} \tag{10}$$
  Summing over all households, we get the demand for the product of firm j (ignoring an irrelevant constant)
$$Q_j = Y \left(\frac{P_j}{P}\right)^{-\eta} \tag{11}$$
  where
  - Y is the income real aggregate
  - P is the price index of goods, defined in (5)
  - η is the demand elasticity
- The firm maximizes profits
$$\pi_j \equiv P_j Q_j - W L_j \tag{12}$$
  s.t. (9) and (11).
  The solution is
$$\frac{P_i}{P} = \frac{\eta}{\eta - 1} \frac{W}{P} \tag{13}$$
  The assumption $\eta > 1$ garantees that there exists a price that maximizes profits.
- The real price of the good is equal to marginal cost,
  plus a profit margin ("markup": margin of price over marginal cost)
- More monopoly power means a lower η, and a higher margin.
  With perfect competition, $\eta = \infty$ and $P_i = W$.

Aggregate Demand

to determine the level of prices, we use the following function of **aggregate demand:**

$$P \cdot Y = M \tag{14}$$

where M is the money supply.

The money supply determines nominal demand.

(14) can be justified, for example, with a model in which real money balances enter into the utility function (Blanchard/Kiyotaki, 1987).

Equilibrium

- The model is symmetric: all the firms are identical, have the same monopoly power and the same preferences. If there are as many firms as households, $\pi_i = \pi_j$ in symmetric equilibrium.
- Therefore in equilibrium, each one produces the same:

$$L_i = Q_i = Y \ \forall i \tag{15}$$

- Substituting (15) in (8):

$$Y = \left(\frac{W}{P}\right)^{\frac{1}{\gamma-1}} \tag{16}$$

$$\rightarrow \frac{W}{P} = Y^{\gamma-1} \tag{17}$$

- Substituting (17) in (13):

$$\frac{P_i}{P} = \frac{\eta}{\eta-1} Y^{\gamma-1} \tag{18}$$

  In equilibrium, the prices of all the goods are equal:

$$P_i = P \ \forall i \tag{19}$$

- From (18) we get

$$Y = \left(\frac{\eta-1}{\eta}\right)^{\frac{1}{\gamma-1}} \tag{20}$$

- From(20) and (14) we get

$$P = \frac{M}{\left(\frac{\eta-1}{\eta}\right)^{\frac{1}{\gamma-1}}} \tag{21}$$

3.2 Implications of the model

I Price > marginal cost,

therefore the firms are willing to produce more if there is an unexpected increase in demand.

II The production in equilibrium is smaller than the social optimum.

The social optimum is obtained maximizing (1), s.t.
$C_i = \overline{Y} = \overline{L} = L_i$:

$$\max_{\overline{L}} \overline{L} - \frac{1}{\gamma}\overline{L}^{\gamma}$$

First order condition:

$$1 - \overline{L}^{\gamma-1} = 0$$

which implies

$$L^{OptSocial} = Y^{OptSocial} = 1 \tag{22}$$

$$> Y = \left(\frac{\eta - 1}{\eta}\right)^{\frac{1}{\gamma-1}} \tag{23}$$

I and II together imply that an increase of production, (as a consequence of a monetary shock and incomplete adjustment of prices, for example) has the advantage of reducing the difference between actual production and optimal production.

III **Externality of aggregate demand**

- Suppose that, for whatever reason all the firms reduce the price of their product by ϵ.
- Then the level of prices goes down, and the aggregate demand $\left(\frac{M}{P}\right)$ and the production rise.
- Then the production of all firms increases; they are willing to produce more, because the price is greater than marginal cost.
- Social welfare increases, because production was lower than the social optimum.

IV **Monetary neutrality**

Imperfect competition is compatible with monetary neutrality.

An increase of the quantity of money makes the price index and the wage increase in the same proportion, and the production does not change.