On the relevance of saving for labor market fluctuations*

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Abstract

This paper studies how the labor market dynamics of the Mortensen-Pissarides matching model change when concave preferences and saving are considered. We compare (1) the standard model where all agents have linear utility, so the interest rate is constant; (2) a model where capitalists have concave utility, so interest rates vary; (3) a model where both capitalists and workers have concave utility, but workers have no access to capital markets; (4) a model where workers engage in precautionary saving. Taking into account worker risk aversion has a strong effect on the wage bargaining game, and thus on labor market dynamics, when workers have no access to capital markets. However, when precautionary saving is allowed, labor market dynamics are much closer to those of the standard linear-utility setup, in spite of the fact that workers do not save very much in our calibration.

Calculating the precautionary saving model requires us to compute the equilibrium dynamics of the asset distribution. We follow the method of Reiter (2006), calculating a steady state on a fine grid of asset levels, and then linearizing the solution over this same grid with respect to aggregate shocks. The dynamics caused by productivity shocks can be approximated fairly well by tracking a few aggregate statistics, as in Krusell and Smith (1998). But when shocks have important redistributional consequences, accuracy requires a higher-dimensional representation of the equilibrium.

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1 Introduction

Labor market dynamics, and especially changes in the unemployment rate, play a central role in debates about the importance of business cycles both in academic economics and in the popular press. Partly for this reason, models of unemployment over the business cycle are a hot topic of current research. It is of particular interest to investigate unemployment fluctuations in models where — unlike the Mortensen/Pissarides baseline model — workers are risk averse and are imperfectly insured against unemployment. However, owing to the technical difficulty of characterizing distributional dynamics in general equilibrium, economists are only now beginning to address models of this type.

In this paper we propose a benchmark model of the labor market where workers engage in precautionary saving, and we solve its business cycle dynamics using a highly accurate but tractable algorithm. We start from the most standard Mortensen/Pissarides model and go via several steps to a model with physical capital where both capitalists and workers are risk averse savers. Our focus throughout is on how incorporating saving into the matching model affects the dynamics of labor market aggregates. Of course, there are many dimensions for which it might be important whether workers can save or not. For example, the effect of government finance on real economic variables depends on how much households deviate from Ricardian behavior, and the welfare cost of business cycles will also depend on workers' saving possibilities. But these issues are beyond the scope of the paper.

We focus on a specification which seems to us the simplest case applicable to the issues we address. In our model, workers must match with vacancies, created by capitalists, in order for production to occur. Physical capital is also required for production; there are no adjustment costs in changing capital. Matching is random; both workers and vacancies are identical *ex ante*; separation is exogenous. Business cycles are driven by aggregate technology shocks. Wages are determined by Nash bargaining, relative to the threat of separation, on a short term (one period) basis. Since we are concerned with documenting the effects of saving, we explicitly take into account the quantitatively important effect of asset holdings on the wage bargaining game. We calibrate the model so that the median worker's assets are only a small part of the *per capita* capital stock.

Characterizing the cyclical behavior of the labor market requires us to solve for the dynamics of the wealth distribution over time. For the sake of accuracy, we employ the algorithm of Reiter (2006). This begins with a precise nonlinear calculation of the steady state equilibrium asset distribution on a fine grid. It then linearizes the dynamics on this grid with respect to aggregate shocks. In addition to studying cyclical dynamics, this method permits us to calculate the effects of wealth redistributions. We can also investigate to what extent the high-dimensional aggregate dynamics can be approximated by conditioning on a small set of moments, as in Krusell and Smith (1998).

As we hinted in the second-to-last paragraph, incorporating savings into the matching framework involves many specification choices. To begin with, the model must specify what type of assets are available to workers for saving. The model must either incorporate a single asset distribution across one class of agents who are both workers and entrepreneurs, or must incorporate different types of agents to play these roles. Different matching structures are possible, such as random matching equilibrium or competitive search equilibrium. The bargaining game must be specified, including the relevant threat point, possible interactions between workers, and the effects of ownership of physical capital (by the firm) or savings (by the worker). The bargaining game could determine the wage over a single period, or it could determine a long-term contract. Heterogeneity of workers or jobs could be included, possibly as a way of endogenizing separation.

The specification we study here is chosen primarily for simplicity, not because we are sure it is the best one. The growing literature on matching models with risk averse agents has considered a variety of different setups, and we believe this is a good thing, because there are so many possibilities that ought to be explored. A number of papers have studied unemployment insurance in steady state models of matching and precautionary saving, including early contributions by Costain (1999) and Acemoglu and Shimer (2000), and more recent treatments by Reichling (2006) and Roca (2007) using newer computational methods. Several papers have studied the cyclical dynamics of matching models where workers are risk averse but are assumed unable to save, including Beaudry and Pages (2001) and Rudanko (2006). The latter paper considers long-term wage contracting and also compares competitive search equilibrium to random matching equilibrium.

The papers most closely related to this one are business cycle models with labor market matching in which workers protect themselves against unemployment risk through precautionary saving. To date several papers have attempted this, including Costain and Reiter (2005), Shao and Silos (2007), Nakajima (2007), and Krusell, Mukoyama, and Sahin (2007). All these papers, like this one, study random matching and short-term Nash wage bargaining. All except that of Krusell *et al.* (2007) simplify the role of individual savings in the wage bargaining problem by assuming that unions negotiate on workers' behalf. While Nakajima (2007) and Krusell *et al.* (2007) assume there is just one class of agents, who own all capital and firms and also act as workers, the other papers treat entrepreneurs and workers as separate types. Except for our own previous work, all these papers solved the distributional dynamics on the basis of some variant of the algorithm of Krusell and Smith (1998).

2 Model

2.1 Overview

The purpose of this section is to derive a model that allows capital accumulation of workers, but is otherwise as simple as possible, and as close as possible to a standard Mortensen/Pissarides matching model.

We want to highlight two important modeling choices. First, we assume that there are two types of agents, capitalists and workers. This assumption serves several purposes. It makes it easy to calibrate the model with a realistic aggregate capital/output ratio, while nevertheless ensuring that the median worker holds assets equivalent to just a few months of wage income.[1] Furthermore, all firms (matches) are owned by capitalists, who will be assumed identical. This guarantees that the firm's problem is well-defined, because the entrepreneurs' marginal utility defines the stochastic discount factor. Finally, since workers are not allowed to trade in firms, only one asset is traded in equilibrium, namely physical capital. Therefore there is no need to solve a portfolio choice problem.

The second important modeling choice is that we restrict the wage contracts to be short term, that is, valid for only one period. Moreover, we assume that each period's wage bargaining takes place between an individual firm and an individual worker. This kind of contract leaves important efficiency gains unexploited. First, since capital markets are incomplete, there is room for an intertemporal insurance contract between firms

[1] In Nakajima (2007) and Krusell, Mukoyama, and Sahin (2007), which have only one class of agents, a typical worker holds several years of income, implying much greater self-insurance. Clearly, additional heterogeneity could be introduced into their models (in discount factors, for instance) to spread out the distribution of wealth, endogenously sorting agents into some who earn mostly from labor and others who earn mostly from investment. But for present purposes, sorting agents exogenously into two classes seems a simpler and clearer way of achieving a realistic wealth distribution.

and workers. Second, short term contracts give rise to a strategic savings motive for workers, who accumulate assets to improve their bargaining position in future wage negotiations. This induces overaccumulation of assets, especially by workers who currently hold few assets. Neither this short-term contracting nor perfectly efficient long-term contracts look very realistic. We adopt this contracting environment because we want to take seriously the possible effects of saving on the bargaining problem, and modeling this inside a long-term contract appears intractable at the moment.

2.2 Timing

The model is written in discrete time. The sequence of events in each period is the following:

1. The aggregate productivity shock is realized
2. Firms post vacancies
3. Matching takes place
4. Wage is negotiated for the current period
5. Firms rent capital and produce
6. Households consume
7. Job separation shocks are realized

2.3 Production Technology

The output Y_{ti} of a match (a single-worker firm) indexed by i at time t is given by

$$Y_{ti} = F(k_{ti}, z_t) = z_t A k_{ti}^{\alpha} \tag{1}$$

where z_t is an aggregate productivity factor and A is a constant. Physical capital can be adjusted frictionlessly by the firm, after negotiating the wage. This means the firm chooses capital by solving the static problem

$$\max_k F(k, z_t) - (r_t + \delta)k \quad = \quad z_t^{\frac{1}{1-\alpha}} A \left(\frac{\alpha}{r_t}\right)^{\frac{\alpha}{1-\alpha}} \tag{2}$$

with first order condition

$$F_k(k_{ti}, z_t) = r_t + \delta \tag{3}$$

Notice therefore that as long as capital can be adjusted without frictions, and agents have linear utility, the labor market dynamics of a matching model with capital are equivalent to those of a model without capital, except that the percentage variation of the productivity shock is rescaled by the factor $(1-\alpha)^{-1}$. Differences arise if savers have concave utility, because then the interest rate varies in (2). Our numerical work will study the sign and size of these interest rate effects. Difference would also arise if we considered lags in the adjustment of capital, which would lead to a holdup problem, but these issues are beyond the scope of this paper.

The product of labor Y_{ti}^{lab} (both marginal and average) is what is left after the rental cost of capital:

$$Y_{ti}^{lab} = (1-\alpha) z_t A k_{ti}^{\alpha} \tag{4}$$

This has to be split between firm and worker, according to the bargaining outcome described in Section 2.7. We assume that aggregate productivity follows a first order autoregressive process in logs:

$$\log z_{t+1} = \rho_z \log z_t + \epsilon_{t+1} \tag{5}$$

2.4 Workers and entrepreneurs

In this economy, there are two types of assets: capital and shares of firms. Firms that currently have a worker have a positive value, because they earn part of the match surplus.

The are also two types of households, entrepreneurs and workers. These two types of households have the same preferences, but we will assume they have different opportunities in financial markets. Entrepreneurs own the firms and they can also own capital. They have no labor endowment and therefore suffer no idiosyncratic shocks, so that we can talk about a representative entrepreneur in the economy. Since entrepreneurs are the only households that are allowed to own firms, and since they are all alike, we can assume without loss of generality that firms cannot be traded. The capital holdings of entrepreneurs then follow the dynamic equation

$$K_t^f = (1 + r(t))K_{t-1}^f + \pi_t - C^f(t) \tag{6}$$

where π_t is the cash flow that entrepreneurs receive from the firms in period t. Their return on their capital holdings, $r(t)$, is the marginal product of capital. The only decisions that entrepreneurs make is how much to save in capital. The corresponding Euler equation is

$$U'(C^f(\Omega)) = \beta\, \mathrm{E}_{\Omega'} \left(1 + r(\Omega')\right) U'(C^f(\Omega'))$$

Workers cannot own firms, they can only own capital. In order to obtain a realistic calibration of the capital holding of workers, we introduce the following capital market friction. We assume that workers do not obtain the full net marginal product of capital.[2] They have to pay an intermediation (or agency) cost ϕ, such that $r^w(t) = r(t) - \phi$. The capital holdings of employed households evolve according to

$$\dot{k} = r^w(\Omega)k + w\,(k;\Omega) - C^e_\Omega(k) \tag{7}$$

For the unemployed we have

$$\dot{k} = r^w(\Omega)k + b - C^u_\Omega(k) \tag{8}$$

In the frictionless case $\phi = 0$, we know that $r^{w*} = r^* < \beta^{-1} - 1$ because of precautionary saving. In this case, entrepreneurs are driven out of the capital market in the deterministic steady state, since they have no precautionary savings motive. Then their only assets are firms, which they cannot trade with workers.

2.5 Matching Technology

The unemployment rate at the beginning of period t, which here is simply the number of unemployed workers, is denoted by U_t. Employment $(1-U_t)$ changes over time according to

$$(1 - U_{t+1}) = (1 - \sigma)(1 - U_t) + M_t \tag{9}$$

Here σ is the exogenous rate at which existing matches are destroyed. New job matches M_t are created according to the matching function

$$M_t = \mu U_t^\lambda V_t^{1-\lambda} \tag{10}$$

where V_t is the number of open vacancies, and μ is a constant. Defining labor market tightness as $\theta = \frac{V_t}{U_t}$, we can write the probability of a firm to fill the vacancy as ${p^F}_t = \frac{M_t}{V_t} = \mu\theta^{-\lambda}$ and the probability of a worker to find a job as $p_t^W = \frac{M_t}{U_t} = \theta {p^F}_t = \mu\theta^{1-\lambda} = \mu^{1/\lambda} {p^F}_t^{(\lambda-1)/\lambda}$.

[2]Alternatively one could assume that workers are more impatient. We prefer a specification where workers and entrepreneurs differ in the constraints that they face, not in their preferences.

2.6 Value functions

The solution of the model can be characterized by the value functions of workers and firms. Let $V^e(k;\Omega)$ and $V^u(k;\Omega)$ denote the value functions of an employed and an unemployed worker, respectively. The value function of the firm is the value of a filled vacancy, denoted by $J(k;\Omega)$. The values are function of the current aggregate state, which we will characterize later, and the capital holding of the worker. The worker's level of capital affects the value of the firm through the wage bargaining, cf. Section 2.7. The value functions satisfy

$$\begin{aligned}
V^e(k;\Omega) &= \max_c \Big\{ U(c) + \beta\, \mathrm{E}_{\Omega'} \Big[(1-\sigma) V^e\big((1+r^w(\Omega))k + w(k;\Omega) - c;\Omega'\big) \\
&\qquad + \sigma V^u\big((1+r^w(\Omega))k + w(k;\Omega) - c;\Omega'\big) \Big] \Big\} && (11)\\
V^u(k;\Omega) &= \max_c \Big\{ U(c) + \beta\, \mathrm{E}_{\Omega'} \Big[p^W(\Omega') V^e\big((1+r^w(\Omega))k + b - c;\Omega'\big) \\
&\qquad + (1-p^W(\Omega')) V^u\big((1+r^w(\Omega))k + b - c;\Omega'\big) \Big] \Big\} && (12)\\
J(k;\Omega) &= U'(C^f)\Big(Y_t^{lab} - w(k;\Omega)\Big) + \beta(1-\sigma)\, \mathrm{E}_{\Omega'}\, J\big(k^e(k,w;\Omega);\Omega'\big) && (13)
\end{aligned}$$

As defined in (4), Y_t^{lab} is the match output after subtracting capital costs. The fact that there is a representative entrepreneur makes the value of the firm in (13) well-defined: all firms discount their cash flow with the marginal utility of this entrepreneur.

It is instructive to look at the Euler equation of the households in this model. Let us abbreviate by $C^e(k;\Omega) \equiv C^e((1+r^w(\Omega))k + w;\Omega)$ and $C^u(k;\Omega) \equiv C^u((1+r^w(\Omega))k + w;\Omega)$ the optimal consumption choices of an employed and an unemployed worker, respectively, where wage and interest rate are a function of the state variables Ω and k. Similarly, $k^e_\Omega(k) \equiv (1+r_\Omega)k + w(k;\Omega) - C^e(k;\Omega)$ and $k^u_\Omega(k) \equiv (1+r_\Omega)k + b - C^u(k;\Omega)$ denote the corresponding savings functions. From the envelope conditions

$$\dot V^e_\Omega(k) = U'(C^e(k;\Omega))(1 + r_\Omega + w'_\Omega(k)) \tag{14}$$
$$\dot V^u_\Omega(k) = U'(C^u(k;\Omega))(1 + r_\Omega) \tag{15}$$

we get the household Euler equations:

$$\begin{aligned}
U'(C^e(k;\Omega)) &= \beta\, \mathrm{E}_{\Omega'} \Big[(1-\sigma)\Big(1 + r^w(\Omega') + \frac{\partial w(k;\Omega')}{\partial k}\Big) U'(C^e(k^e(\Omega,k);\Omega')) \\
&\qquad + \sigma\big(1 + r^w(\Omega')\big) U'(C^u(k^e(\Omega,k);\Omega')) \Big] && (16)\\
U'(C^u(k;\Omega)) &= \beta\, \mathrm{E}_{\Omega'} \Big[p^W(\Omega')\Big(1 + r^w(\Omega') + \frac{\partial w(k;\Omega')}{\partial k}\Big) U'(C^e(k^u(\Omega,k);\Omega')) \\
&\qquad + (1-p^W(\Omega'))\big(1 + r^w(\Omega')\big) U'(C^u(k^u(\Omega,k);\Omega')) \Big] && (17)
\end{aligned}$$

The reward to saving has two components in this model: the interest rate $r^w(\Omega')$, and the increased wage that the worker will be able to bargain next period if she has a better outside option due to higher assets. We will show later that for workers with few assets, the last component is much more important than the interest rate. Therefore, in steady state there will be very few workers with very low wealth holdings. This increased savings motive is a consequence of the inefficient contracting environment. Only short-term contracts are available, therefore households make a strong effort to save and improve their bargaining position in future periods. An efficient long-term contract between worker and employer would specify both wages and consumption, to avoid this strategic behavior on the part of the worker.

2.7 Bargaining

Workers and firms share the surplus from the match. To define the surpluses, let us first write the value and consumptions functions such that the dependence on the wage w explicit. The worker's value, *conditional on the current wage*, is defined as

$$\tilde{V}^e(k,w;\Omega) = U(C^e(k,w;\Omega)) + \beta \, \mathrm{E}_{\Omega'} \left[(1-\sigma)V^e\left(k^e(k,w;\Omega);\Omega'\right) + \sigma V^u\left(k^e(k,w;\Omega);\Omega'\right)\right]$$

Similarly, the firm's value, *conditional on the current wage*, is

$$\tilde{J}(k,w;\Omega) = U'(C^f(\Omega))(MPL - w) + (1-\sigma)\, \mathrm{E}_{\Omega'}\, J\left(k^e(k,w;\Omega);\Omega'\right) \tag{18}$$

The Nash bargaining wage maximizes a weighted product of the worker's and the firm's surpluses:

$$w(k) = \underset{w}{\operatorname{argmax}} \left(\tilde{V}^e(k,w;\Omega) - V^u\left(k;\Omega\right)\right)^{\alpha} \tilde{J}(k,w;\Omega)^{1-\alpha} \tag{19}$$

where α denotes the relative bargaining power of workers. In some of our simulations, we will assume α is stochastic. If so, it will be an AR(1) process in logs, just like the technology shock:

$$\log(\alpha_{t+1}/\alpha^*) = \rho_\alpha \log(\alpha_t/\alpha^*) + \epsilon^\alpha_{t+1} \tag{20}$$

The FOC for the bargaining problem is

$$\frac{\tilde{V}^e(k,w;\Omega) - V^u\left(k;\Omega\right)}{\alpha \frac{\partial \tilde{V}^e(k,w;\Omega)}{\partial w}} = -\frac{\tilde{J}(k,w;\Omega)}{(1-\alpha)\frac{\partial \tilde{J}(k,w;\Omega)}{\partial w}} \tag{21}$$

From the envelope condition of the household problem we know that

$$\frac{\partial \tilde{V}^e(k,w;\Omega)}{\partial w} = U'(C^e(k,w;\Omega)) \tag{22}$$

Differentiating (18) we get

$$\frac{\partial \tilde{J}(k,w;\Omega)}{\partial w} = -U'(C^f(\Omega)) + (1-\sigma)\left(1 - \frac{\partial C^e(k,w;\Omega)}{\partial w}\right) \mathrm{E}_{\Omega'}\, J_k\left(k^e(k,w;\Omega);\Omega'\right) \tag{23}$$

Although employed workers are never liquidity constrained in our simple model, the fact that unemployed households may be liquidity constrained creates a serious technical difficulty with the wage bargaining condition. Denote by $k_c(\Omega))$ the point where the constraint starts binding for the unemployed. For low levels of k, an employed household will not save enough to end up with more than $k_c(\Omega')$, and therefore there is a k_0 such that $k^e(k_0,w;\Omega) = k_c(\Omega')$. The consumption decision of the employed is connected to the consumption of the unemployed through the Euler equation (16). From that we see that $C^e\left(\Omega;k\right)$ has a kink at $k = k_0$. This means that $\frac{\partial C^e(k,w;\Omega)}{\partial w}$ has a discontinuity at $k = k_0$ (with w equal the equilibrium wage). which enters the rhs of (21) through (23), and induces a discontinuity in the wage function, then in the firm's value function, etc. Discontinuities in the wage and value functions cannot be handled for at least two reasons. First, our computational approach is based on smooth approximation. Second, discontinuities destroy the convexity of the household problem.

A natural way to avoid this problem is to go to the continuous time limit. Notice that the liquidity constraint kicks in at a level of assets roughly equal to one period of labor income. If the time period is one minute, it appears at an asset level of about 10 cents. In the continuous time limit, the kink disappears. That means, the numerical error from ignoring the kink is negligible if the time period is very small. In the limit, $\frac{\partial C^e(k,w;\Omega)}{\partial w} \to 0$, and $k^e(k,w;\Omega) \to k$. Then (21), using (22) and (23), reduces to

$$\frac{V^e(k;\Omega) - V^u(k;\Omega)}{\alpha U'(C^e(k;\Omega))} = \frac{J(k;\Omega)}{(1-\alpha)[U'(C^f(\Omega)) - J_k(k;\Omega')]} \tag{24}$$

We will set the time period of the model to 1/32nd of a week, just over one hour of work. The error from using the continuous time limit (24) in the wage bargaining is then negligible.

2.8 Closing the model

Denote by $\Phi_t^e(k)$ and $\Phi_t^u(k)$ the end-of-period cross-sectional distribution function of capital for employed and unemployed workers, respectively. The are scaled such that U_t, the unemployment rate in period t after matching took place, is given by

$$U_t = \Phi_t^u(\infty) = \int_{\underline{k}}^{\infty} 1 \, d\Phi_t^u(k) \tag{25a}$$

$$1 - U_t = \Phi_t^e(\infty) = \int_{\underline{k}}^{\infty} 1 \, d\Phi_t^e(k) \tag{25b}$$

The zero-profit condition for vacancy creation is

$$\kappa = p^F(\Omega) \int_{\underline{k}}^{\infty} J(k;\Omega) \, d\Phi_{t-1}^u(k) \tag{26}$$

where κ is the flow cost of keeping a vacancy open. The firms are matched in period t against the pool of workers who were unemployed at the end of $t-1$. Note the firms can find a worker and start producing in the same period when the post a vacancy.

The aggregate capital stock at the end of period $t-1$, is given by

$$K_{t-1} = \int_{\underline{k}}^{\infty} k \, d\Phi_{t-1}^e(k) + \int_{\underline{k}}^{\infty} k \, d\Phi_{t-1}^u(k) + K_{t-1}^f \tag{27}$$

This will be used for production in period t. The profits (or cash flow) from firms can now be written as total production Y_t minus the remuneration of capital, the total wage bill $\int_{\underline{k}}^{\infty} W_t(k) \, d\Phi_k^e()$

$$\pi_t = Y_t - r(t)K_{t-1} - \int_{\underline{k}}^{\infty} W_t(k) \, d\Phi_t^e(k) - \kappa V_t \tag{28}$$

The details of the distributional dynamics resulting from the consumption decisions of entrepreneurs and employed and unemployed workers are spelled out in Appendix A.

3 Parameter values

The US and the big European countries differ substantially in their average unemployment rates, and even more in median unemployment durations. They also display large differences in median asset holdings. All these facts are relevant for our model, since the effectiveness of self-insurance against unemployment depends on unemployment duration relative to the typical stock of liquid asset holdings. We therefore investigate several parameter combinations, which include a calibration to US data, a calibration to the most recent German data, and a hybrid 'worst case' that combines German unemployment with US asset levels. We think addressing a potential worst case is useful, because we want to know whether there are any realistic circumstances under which saving behavior would have nontrivial implications for labor market dynamics.

Common parameters

We first describe the parameters that are common to both the US and German calibrations. The model period is set to 1536^{-1} years, where $48 \times 32 = 1536$, so that a period represents slightly more than one hour of production. The time discount rate is set to 5% annually, so $\beta = 0.95^{-1536}$. Depending on the specification considered, capitalists and workers may have linear utility, or the following CRRA utility function:

$$U(c) = \begin{cases} \frac{c^{1-\gamma}-1}{1-\gamma} & \text{if } \gamma \neq 1 \\ \log(c) & \text{if } \gamma = 1 \end{cases} \tag{29}$$

When both capitalists and workers have CRRA utility (the cases MPCRRA and WSav, defined below), we use the same parameter γ for both.

For the benefit parameter we use $b = 0.4$. This is lower than what Costain and Reiter (2007) advocate, but we think that a lower value is interesting in the present context, to make the utility impact of unemployment more severe. For the elasticity of matches with respect to labor market tightness we choose 0.4159, which means the elasticity of matches to unemployment is $\lambda = 0.5841$. This parameter was estimated from a regression of the detrended log of job finding probability on detrended tightness.[3] Workers' bargaining share α is recalibrated in each case we simulate so that on average, the wage is 0.96 times labor productivity. This level of wages suffices so that productivity fluctuations cause the observed level of unemployment variation (Hagedorn and Manovskii 2005) without further elaborating the model to include elements like wage stickiness or embodied technical change. Varying this calibration to 0.95 or 0.97 does not change any important result here.

The capital share in production is set to $\alpha = 1/3$. We target a steady state capital-output ratio of 3 years ($KOR \equiv \frac{K}{Y} = 3 \times 1536$), a marginal product of capital of $r^* = 1/\beta - 1$, and a labor productivity of 1. This is achieved by setting $\delta = \frac{\alpha}{KOR} - r^*$, choosing the capital-labor ratio as $KLR = \frac{1}{r^*+\delta}\frac{\alpha}{1-\alpha}$, and $A = \frac{1}{(1-KLR)^\alpha}$.

The autoregressive parameter for fluctuations in TFP and bargaining power are both set to 0.95 quarterly: $\rho_z = \rho_\alpha = 0.95^{4/1536}$.

US calibration

Following recent papers in the matching literature, we assume a job separation rate of 40% annually, therefore $\sigma = 0.4/1536$. The vacancy cost parameter is set such that the steady state unemployment rate is $U^* = 5.67$ percent, the average in the US in the period 1951–2003. From that we get the steady state job finding probability as $p^{W*} = \sigma \frac{1-U^*}{U^*}$. We can normalize the matching efficiency to $\mu = 1$. This parameter only scales the absolute number of vacancies, and the firms' probability of finding a worker, which are irrelevant for our purpose.

We calibrate the model such that the mean asset holdings of workers equal the financial assets of the US household with median net wealth. From Bertaut and Starr-McCluer (2001, Table 2), we see that median net wealth in the US in 1998 was \$71100. In Table 5 we see that total financial assets of the second net-wealth quintile are the fraction 19.4/53.0 of net wealth, or \$26053. In 1998, the U.S. Bureau of the Census estimates there were 102528 households. GDP in 1998 was \$8747 trillion, which gives a GDP of \$85313 per household. Therefore we estimate median financial assets of households as

[3] The sample period is 1951:1-2004:4. Tightness is measured as help wanted advertisement (HELPWANT) divided by civilian unemployment age 16+ (UNEMPLOY); the acronyms are the identifying codes at FRED. The series for job finding probability was constructed by Robert Shimer and is available on his website, http://robert.shimer.googlepages.com/flows. All data are detrended by HP filter with smoothing parameter 100000.

26053/85313=0.3054 years of GDP per household. We thus choose the intermediation cost ϕ so that in the steady state, the mean asset holdings of a worker are 0.3054 years of GDP per capita. This is achieved by setting the cost to $\phi = 0.693$ annually, which seems perfectly realistic. A problem is that the model does not say what is GDP per capita, because it does not determine the number of capitalists compared to the number of workers. But since capitalists are homogenous, all the capitalists' assets could be held by a single individual, as long as it behaves competitively. We therefore stipulate that the mass of capitalists is zero, and GDP per capita equals GDP per worker.

German calibration

For separation rates or unemployment duration, we do not have historical statistics available for Germany. We therefore use the most recent data. For workers who became unemployed in 2006, the average unemployment duration was 41.4 weeks. We therefore set the job finding probability to $p^W = \frac{41.4}{52}/1536$. The unemployment rate was $U^* =$ 10.7 percent. This gives a separation rate of $\sigma = \frac{p^W U^*}{1-U^*} = 0.0954/1536$. This means that the separation rate in Germany is only one fourth of what it is in the US.

Median net wealth in Germany 1993 was 120188 DM (Boersch-Supan and Eymann 2001, fn. 36). From Table 7 we see that financial assets (minus life-insurance and long-term savings contracts) are 64.4 percent of net worth (weighted average of 64.3 percent in West Germany, and 64.8 percent in East Germany) which gives 77521DM. The number of households in the microcensus 1993 was 36.23 million. GDP in 1993 was 1694.37 billion euros = 3313.9 billion DM. GDP per household was therefore, 91468 DM, so financial assets were 84.75% of annual GDP per household.

The stark contrast between the US and the German calibration may be due to differences in measurement rather than real differences. However, since we want to check the robustness of our results to changes in parameters, these big variations are not unwelcome.

Steady state

In summary, in all simulations certain common and country-specific parameters are fixed as described above. Then for each simulation the workers' bargaining share α and the vacancy cost κ are reset so that the steady state wage is 0.96, and so that steady state unemployment equals its country-specific mean.

4 Results

When we rewrite the Mortensen-Pissarides model with concave rather than linear utility, we are altering several things simultaneously: we are changing the utility functions of capitalists and workers, we are taking a stand on the asset markets open to each class of agent, and we are taking a stand on how asset holdings affect the bargaining game. To untangle the effects caused by each of these changes, we compare four different models. The first is the standard Mortensen-Pissarides model with linear technology and linear utility, denoted in the graphs and tables by MP. In the second model, capitalists have CRRA utility with relative risk aversion γ. Workers still have linear utility, but are excluded from the capital market, so the interest rate varies because it is determined by the consumption smoothing of the capitalists. This model is denoted by MPKap. In the third model, both capitalists and workers are risk averse, with the same degree of relative risk aversion. However, workers are again excluded from the capital market, so they have no way of saving to protect themselves against labor market risk. Therefore workers' risk aversion in this case only alters the model through its effects on the bargaining game. This model is denoted by MPCRRA. Finally, the fourth model is the full model where households are allowed to save, denoted by WSav.

4.1 Dynamics

Table 1 reports moments for the benchmark calibration, when the model is driven either by technology shocks or shocks to workers' bargaining power. Impulse response functions for these two shocks are shown in Figures 3 and 4. As in the paper of Hall (2006), the main message of the results is that all the models have very similar labor market dynamics. Responses to the productivity shock are somewhat more persistent when savers have concave utility, because in these cases, the adjustment of capital is slowed down by the consumption smoothing motive. The effect on capital is strongest in the WSav model, where capital accumulation feeds into the worker's wage, which then reinforces capital accumulation. However, the resulting persistence for output, wages, and unemployment is not very different between the WSav model and the two other cases with variable interest rates. The incentive to adjust capital is not as strong after a bargaining shock (it is caused only by changes in employment, not changes in productivity) so the persistence of the impulse responses varies less with concave utility for these shocks.

The other major differences are found in model MPCRRA, where workers are risk averse and have no access to the capital market. This case gives substantially more variation in vacancies, tightness, and unemployment. It gives less wage variation in response to productivity shocks, and more in response to bargaining power shocks. To understand these findings, note that both workers and capitalists have the same CRRA preferences, with the same value of γ. Nonetheless, workers are effectively more risk averse in the MPCRRA case, because they face uninsured idiosyncratic employment risk, whereas capitalists only face the (much smaller) variation in aggregate productivity. This difference increases workers' expected marginal utility of any given payment of goods, which means the bargaining problem results in a lower wage, *ceteris paribus*. However, for comparability in terms of the observable aspects of the steady state, we reset the worker's bargaining power to maintain the same average wage in each of our simulations. Thus, the MPCRRA case implies a higher worker's share α, making the capitalists' share of surplus smaller and more volatile. Capitalists therefore vary hiring more, increasing unemployment volatility. This is similar to the effect of unemployment benefits on labor market volatility found in Costain and Reiter (2007) and Hagedorn and Manovskii (2005).

4.2 Parameter variations

We explore several other parameter configurations besides the baseline US calibration with logarithmic utility. First, we check the effects of higher risk aversion by increasing γ to 4. Second, we compare our US calibration to a German calibration in which the higher unemployment rate and longer unemployment spells are compensated by higher average asset holdings. Finally, we also explore a hybrid version in which market incompleteness could potentially be more important: an economy with high unemployment like that in Germany, but low asset holdings like those in the US.

Table 2 shows how the simulated moments change when when relative risk aversion increases to $\gamma = 4$, and Figure 5 shows the resulting impulse response functions for a productivity shock. The findings are similar to those in the logarithmic utility case of Fig. 3, except that the increased volatility of labor market flows in the MPCRRA case is amplified, especially in the case of shocks to bargaining power. In other words, the increased effective risk aversion of workers is even more important with $\gamma = 4$ than under logarithmic utility. Similar findings are reported by Rudanko (2006), whose model shares the assumptions of our MPCRRA case, with risk averse workers who are unable to save. Like us, Rudanko reports that increasing risk aversion in this specification stabilizes the wage in the face of productivity shocks, while increasing the volatility of tightness and unemployment, thus partially solving the volatility 'puzzle' of Shimer (2005). Her simulations do not compensate the rise in γ by increasing η to hold fixed the average wage, which suggests that this is not crucial for our results.

Table 3 shows somewhat greater persistence under the German calibration, consistent with the longer unemployment spells in this case. Also, we see that shocks to bargaining power have a substantially larger effect when starting from a calibration with high unemployment. Figures 6 and 7 show the impulse response functions from the hybrid, high-risk calibration with high unemployment and low asset holdings. The effects of technology shocks are almost unchanged from the baseline US calibration. As in the German case, the fact that initial unemployment is high makes bargaining power shocks roughly twice as powerful as in the US baseline of Fig. 4. Still, all qualitative effects in the various parameterizations considered are fundamentally similar to the baseline case.

4.3 Redistributions

Another issue to study in this heterogeneous agent model is the effect of redistributions. Of course, this is only relevant in the WSav setup, since in the other cases considered the workers either have no incentive or no ability to save, and the capitalists, as mentioned before, are effectively a representative agent. In the WSav model, the distribution of assets affects investment, and affects wages, so this is another reason why taking saving into account can potentially have implications for labor market dynamics. Of course, there are many possible ways of redistributing assets, which can have different effects.

Thus, Figure 8 shows how unemployment and capital respond to a variety of changes in the state variables. These impulse responses can be interpreted as the transition paths to steady state if the model starts out of steady state. We only look at variations in initial conditions that represent a wealth redistribution. All in all, we investigate the effect of 8 different redistributions:

- Redistribution from employed to unemployed workers, at the first and 99th percentiles of the respective distributions ("E2U, q=0.01" and "E2U, q=0.99", respectively).
- Redistribution among the unemployed workers, from the workers at percentile 99 to those at percentile 1 ("U, q=0.99 to q=0.01"), and from percentile 75 to percentile 25 ("U, q=0.75 to q=0.25").
- Redistribution among the employed workers, from the workers at percentile 99 to those at percentile 1 ("E, q=0.99 to q=0.01"), and from percentile 75 to percentile 25 ("E, q=0.75 to q=0.25").
- Redistribution from capitalists to employed workers, at the first and at the 99th percentile of the distribution of employed workers ("F2E, q=0.01" and "F2E, q=0.99", respectively).

In each case, the size of the redistribution is scaled so as to be equivalent to 1 percent of annual GDP. Notice that the scaling is only a device to help with quantitative interpretation; a redistribution of this amount is not really feasible, since not all the corresponding groups actually have that many assets.

In the left panels of Fig. 8, we see that by far the most powerful redistributions are those to the poorest unemployed. These redistributions raise unemployment dramatically by raising the outside option of these workers, making them much more expensive to hire. Since wages and unemployment are rising, capitalists have a strong incentive to decrease investment. Also, the new owners of the redistributed capital have a much higher marginal propensity to consume. For both reasons, there is a large and persistent fall in capital. The graph shows that it does not matter very much from whom this transfer is taken. Transfers from the employed to the richest unemployed, on the other hand, have little effect on the labor market.

The right panels show that transfers from the richest to the poorest employed also raise unemployment and drive down capital accumulation, though the effects are an order of magnitude smaller than those seen in the left panels. The mechanism is the same: outside options and wages are raised substantially for those workers who would be the cheapest to hire, and capital is redistributed to agents with a lower propensity to save.

Finally, the right panels also show the effects of redistributions from the capitalists to the employed. In this case, the effects differ depending on who among the employed receives the transfer. If the poorest employed agents receive the transfer, then the effects are like those seen already. But the richest employed agents, because of their precautionary saving incentive, have a higher marginal propensity to save than the capitalists. Also, as seen in Fig. 2, giving them extra wealth has little effect on their wages. So this transfer causes saving and investment to rise; the increased capital makes it worthwhile to hire more too, so unemployment falls.

4.4 The relevance of distributional dynamics

The examples in Fig. 8 show that some changes in the distribution of assets can have nontrivial effects on labor market dynamics. This suggests that algorithms for computing distributional dynamics on the basis of low-dimensional representations could potentially be inaccurate. Therefore, Table 4 investigates to what extent current job-finding probability and next period's capital can be predicted by a small number of state variables. Calculating these variables is an essential part of any algorithm, and in particular implementing the Krusell and Smith (1998) algorithm for this model would require a prediction of these variables.

We consider predictions based on the unemployment rate, the exogenous driving force (for K_{t+1}, this means conditioning on both time t and $t+1$ aggregate shocks), and various statistics that characterize the wealth distribution. We provide both the R^2 and the relative standard error "RelStd", defined as the standard deviation of the prediction error, divided by the standard deviation of the first difference of the predicted series. Note that we can calculate these error statistics to high accuracy because our algorithm uses a much higher-dimensional representation of the equilibrium (savings are represented on a grid of 500 points each for the employed and the unemployed). The first four columns of Table 4 refer to the case where the model is driven only by technology shocks, the last four columns to the case with bargaining shocks only.

Unsurprisingly, predictions based only on the aggregate shock and the unemployment rate are very poor. More interestingly, a prediction which also uses aggregate capital (third row of the table) still achieves a low level of accuracy, with an error that is almost always greater than 10% of the variation that is being predicted (i.e. RelStd$\geq 10^{-1}$). Decomposing aggregate capital into the parts held by capitalists, the employed, and the unemployed often achieves a nontrivial improvement in RelStd, typically by a factor of five or ten when the model is driven by productivity shocks (or for predicting p^W in the bargaining shocks case). But note that even using all five regressors shown in the table, the prediction of capital in the bargaining shocks model is disappointing. None of the specifications shown achieves a relative standard deviation lower than 10%, suggesting that a higher-dimensional representation is essential for accuracy in computing this version of the model.

It is clear why prediction on the basis of a few moments should work better for simulating technology shocks than for bargaining shocks. Technology shocks raise profits and wages simultaneously, so they have little distributional effect on the entrepreneurs and employed workers. Unemployed workers, meanwhile, hold little capital. Therefore, these shocks affect mean capital much more than they affect the shape of the capital distribution. Bargaining shocks, on the other hand, redistribute wealth from entrepreneurs to employed workers. It would therefore be very surprising if mean capital were a sufficient state variable; in fact we see that even knowing the mean capital of each class of agents is far from sufficient when bargaining shocks drive the model. While it is unclear whether bargaining shocks *per se* are empirically interesting, this general point seems likely to remain true for understanding any time-varying redistributional mechanism. Thus high-dimensional simulation methods could well prove crucial for analyzing many fiscal or social welfare policies potentially applicable to unemployment over the business cycle.

5 Conclusions

[TO BE ADDED]

Model	Y	$wage$	MPL	Vac	θ	pW	U
Model driven by technology shock							
Standard deviations							
MP	1.87	1.79	1.82	1.97	3.10	1.29	1.18
MPKap	1.90	1.78	1.83	1.97	3.09	1.29	1.18
MPCRRA	1.91	1.75	1.82	2.44	3.84	1.60	1.47
WSav	1.97	1.87	1.90	1.99	3.15	1.31	1.21
Relative standard deviations							
MP	1.00	0.96	0.98	1.05	1.66	0.69	0.63
MPKap	1.00	0.94	0.96	1.04	1.63	0.68	0.62
MPCRRA	1.00	0.92	0.96	1.28	2.01	0.84	0.77
WSav	1.00	0.95	0.96	1.01	1.60	0.66	0.61
Autocorrelation							
MP	0.90	0.90	0.90	0.85	0.90	0.90	0.93
MPKap	0.90	0.90	0.90	0.85	0.90	0.90	0.93
MPCRRA	0.90	0.90	0.90	0.85	0.90	0.90	0.93
WSav	0.91	0.91	0.91	0.86	0.91	0.91	0.94
Model driven by bargaining shock							
Standard deviations							
MP	0.05	0.12	0.00	2.11	3.32	1.38	1.26
MPKap	0.05	0.14	0.03	2.08	3.27	1.36	1.25
MPCRRA	0.06	0.17	0.03	2.53	3.97	1.65	1.51
WSav	0.05	0.14	0.03	2.09	3.28	1.37	1.25
Relative standard deviations							
MP	1.00	2.31	0.00	41.50	65.24	27.13	24.89
MPKap	1.00	2.73	0.49	40.10	63.04	26.22	24.05
MPCRRA	1.00	2.73	0.48	40.08	63.00	26.20	24.03
WSav	1.00	2.96	0.55	42.83	67.24	27.97	25.62
Autocorrelation							
MP	0.93	0.90	0.97	0.85	0.90	0.90	0.93
MPKap	0.94	0.91	0.93	0.85	0.90	0.90	0.93
MPCRRA	0.94	0.91	0.93	0.85	0.90	0.90	0.93
WSav	0.93	0.91	0.94	0.85	0.90	0.90	0.93

Table 1: Comparison of models, US calibration, $\gamma = 1$

Model	Y	$wage$	MPL	Vac	θ	pW	U
Model driven by technology shock							
Standard deviations							
MP	1.77	1.70	1.73	1.87	2.94	1.22	1.12
MPKap	1.79	1.68	1.73	1.86	2.92	1.22	1.12
MPCRRA	1.85	1.53	1.70	4.26	6.70	2.79	2.56
WSav	1.86	1.72	1.78	2.28	3.58	1.49	1.37
Relative standard deviations							
MP	1.00	0.96	0.98	1.05	1.66	0.69	0.63
MPKap	1.00	0.94	0.96	1.04	1.63	0.68	0.62
MPCRRA	1.00	0.83	0.92	2.30	3.62	1.51	1.38
WSav	1.00	0.92	0.96	1.22	1.92	0.80	0.74
Autocorrelation							
MP	0.90	0.90	0.90	0.85	0.90	0.90	0.93
MPKap	0.90	0.90	0.90	0.85	0.90	0.90	0.93
MPCRRA	0.91	0.90	0.90	0.85	0.90	0.90	0.93
WSav	0.91	0.91	0.91	0.84	0.90	0.90	0.93
Model driven by bargaining shock							
Standard deviations							
MP	0.05	0.12	0.00	2.18	3.44	1.43	1.32
MPKap	0.05	0.15	0.03	2.15	3.40	1.41	1.30
MPCRRA	0.20	0.53	0.10	7.73	12.21	5.08	4.68
WSav	0.05	0.16	0.03	2.20	3.48	1.45	1.33
Relative standard deviations							
MP	1.00	2.30	0.00	40.99	64.78	26.94	24.82
MPKap	1.00	2.69	0.50	39.29	62.09	25.82	23.79
MPCRRA	1.00	2.69	0.49	39.23	61.98	25.78	23.74
WSav	1.00	3.23	0.67	45.75	72.22	30.04	27.67
Autocorrelation							
MP	0.94	0.91	0.97	0.86	0.91	0.91	0.94
MPKap	0.94	0.92	0.94	0.86	0.91	0.91	0.94
MPCRRA	0.94	0.91	0.94	0.86	0.91	0.91	0.94
WSav	0.93	0.93	0.95	0.86	0.91	0.91	0.94

Table 2: Comparison of models, US calibration, $\gamma = 4$

Model	Y	$wage$	MPL	Vac	θ	pW	U
Model driven by technology shock							
Standard deviations							
MP	2.17	2.02	2.10	2.50	3.38	1.41	1.09
MPKap	2.21	1.99	2.10	2.49	3.38	1.41	1.10
MPCRRA	2.23	1.93	2.09	3.19	4.33	1.80	1.40
WSav	2.32	2.11	2.20	2.67	3.66	1.52	1.20
Relative standard deviations							
MP	1.00	0.93	0.97	1.15	1.55	0.65	0.50
MPKap	1.00	0.90	0.95	1.13	1.53	0.64	0.49
MPCRRA	1.00	0.87	0.93	1.43	1.94	0.81	0.63
WSav	1.00	0.91	0.95	1.15	1.58	0.66	0.52
Autocorrelation							
MP	0.93	0.93	0.93	0.88	0.93	0.93	0.98
MPKap	0.93	0.93	0.93	0.89	0.93	0.93	0.98
MPCRRA	0.93	0.93	0.93	0.88	0.93	0.93	0.98
WSav	0.94	0.94	0.93	0.89	0.94	0.94	0.98
Model driven by bargaining shock							
Standard deviations							
MP	0.17	0.32	0.00	4.81	6.48	2.70	2.07
MPKap	0.18	0.39	0.08	4.72	6.35	2.64	2.03
MPCRRA	0.23	0.51	0.10	6.10	8.19	3.41	2.62
WSav	0.16	0.41	0.09	4.87	6.51	2.71	2.05
Relative standard deviations							
MP	1.00	1.87	0.00	28.27	38.06	15.83	12.18
MPKap	1.00	2.24	0.45	26.91	36.19	15.05	11.57
MPCRRA	1.00	2.25	0.45	26.88	36.14	15.03	11.55
WSav	1.00	2.54	0.55	30.01	40.14	16.70	12.64
Autocorrelation							
MP	0.98	0.93	0.92	0.89	0.93	0.93	0.98
MPKap	0.98	0.94	0.98	0.88	0.93	0.93	0.98
MPCRRA	0.98	0.94	0.98	0.88	0.93	0.93	0.98
WSav	0.98	0.95	0.98	0.88	0.93	0.93	0.98

Table 3: Comparison of models, German calibration, $\gamma = 1$

	Productivity shocks				Bargaining shocks			
	p^W		K_{t+1}		p^W		K_{t+1}	
Regressors	$1-R^2$	RelStd	$1-R^2$	RelStd	$1-R^2$	RelStd	$1-R^2$	RelStd
US calibration								
Z,U	5.11e-3	3.49e-1	1.31e-1	7.36e+0	3.09e-4	7.34e-2	2.60e-1	4.52e+0
Z,U,K(U)	6.50e-4	1.24e-1	3.54e-4	3.82e-1	3.08e-5	2.32e-2	7.25e-4	2.39e-1
Z,U,K(total)	6.21e-4	1.22e-1	9.40e-5	1.97e-1	3.72e-5	2.55e-2	4.44e-4	1.87e-1
Z,U,K(total),K(U)	5.77e-4	1.17e-1	4.51e-5	1.37e-1	4.08e-6	8.44e-3	4.43e-4	1.86e-1
Z,U,K(E),K(U),K(F)	8.45e-6	1.42e-2	5.39e-7	1.49e-2	3.22e-7	2.37e-3	4.23e-4	1.82e-1
German calibration								
Z,U	9.47e-3	5.28e-1	1.74e-1	1.08e+1	3.57e-4	6.53e-2	3.72e-1	4.82e+0
Z,U,K(U)	1.66e-3	2.21e-1	1.13e-3	8.72e-1	6.70e-6	8.94e-3	1.43e-3	2.99e-1
Z,U,K(total)	1.21e-3	1.88e-1	3.52e-5	1.54e-1	8.56e-6	1.01e-2	5.62e-4	1.87e-1
Z,U,K(total),K(U)	4.37e-4	1.13e-1	3.50e-5	1.53e-1	4.39e-6	7.24e-3	2.55e-4	1.26e-1
Z,U,K(E),K(U),K(F)	5.47e-5	4.01e-2	7.28e-7	2.21e-2	2.40e-6	5.36e-3	2.15e-4	1.16e-1
High unemployment, low assets calibration								
Z,U	7.29e-4	9.52e-2	4.29e-1	1.43e+1	8.65e-4	9.71e-2	1.52e-1	3.67e+0
Z,U,K(U)	2.42e-4	5.49e-2	4.79e-4	4.79e-1	8.60e-4	9.68e-2	2.40e-2	1.46e+0
Z,U,K(total)	2.47e-4	5.54e-2	1.03e-4	2.23e-1	7.90e-4	9.28e-2	6.79e-4	2.46e-1
Z,U,K(total),K(U)	2.23e-4	5.27e-2	8.10e-5	1.97e-1	7.46e-6	9.02e-3	5.57e-4	2.22e-1
Z,U,K(E),K(U),K(F)	2.07e-4	5.07e-2	7.64e-6	6.05e-2	8.16e-7	2.98e-3	2.50e-4	1.49e-1

Table 4: Prediction error of Krusell-Smith regressions

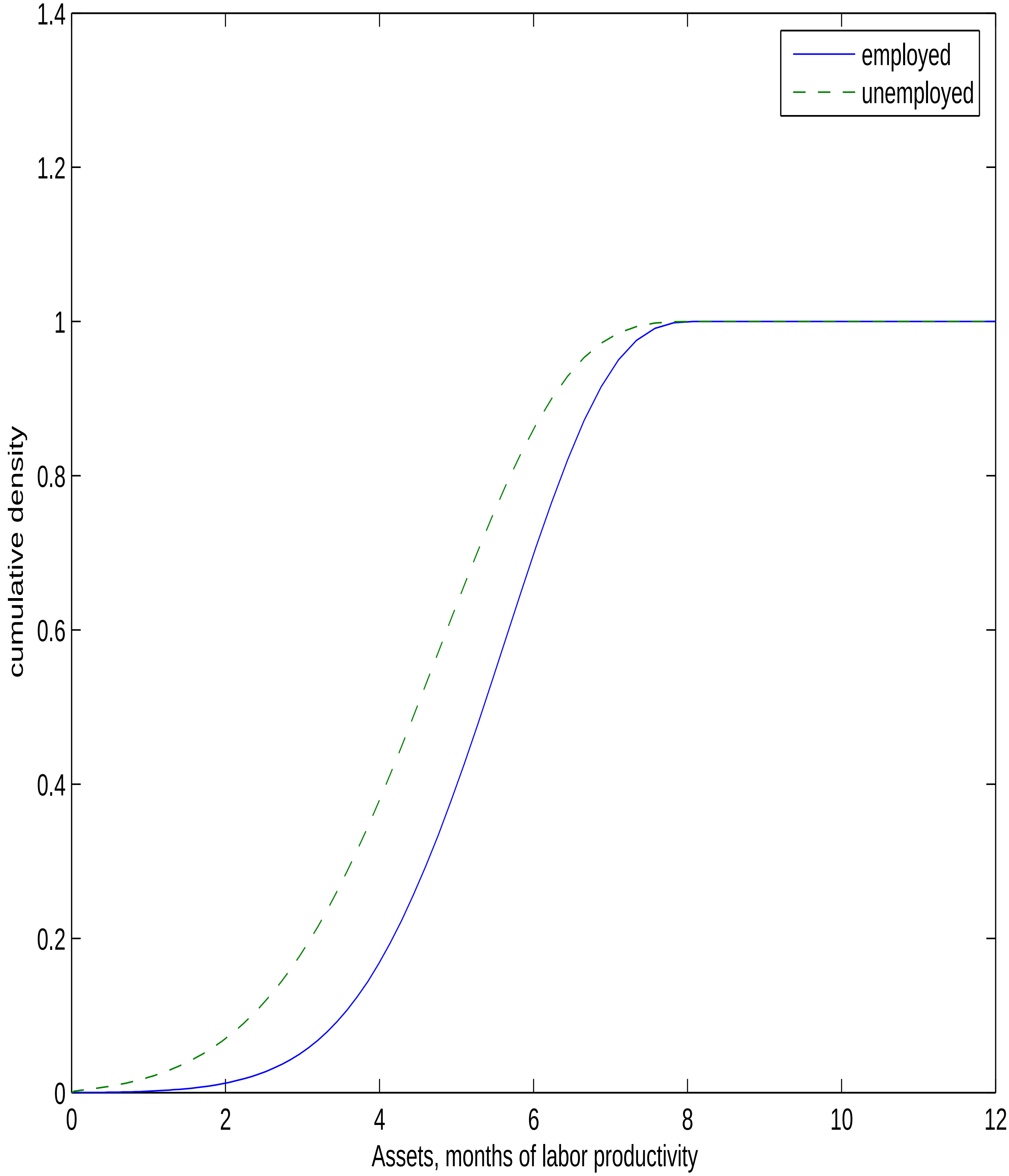

Figure 1: Cumulative density of workers' assets, US calibration, $\gamma = 1$

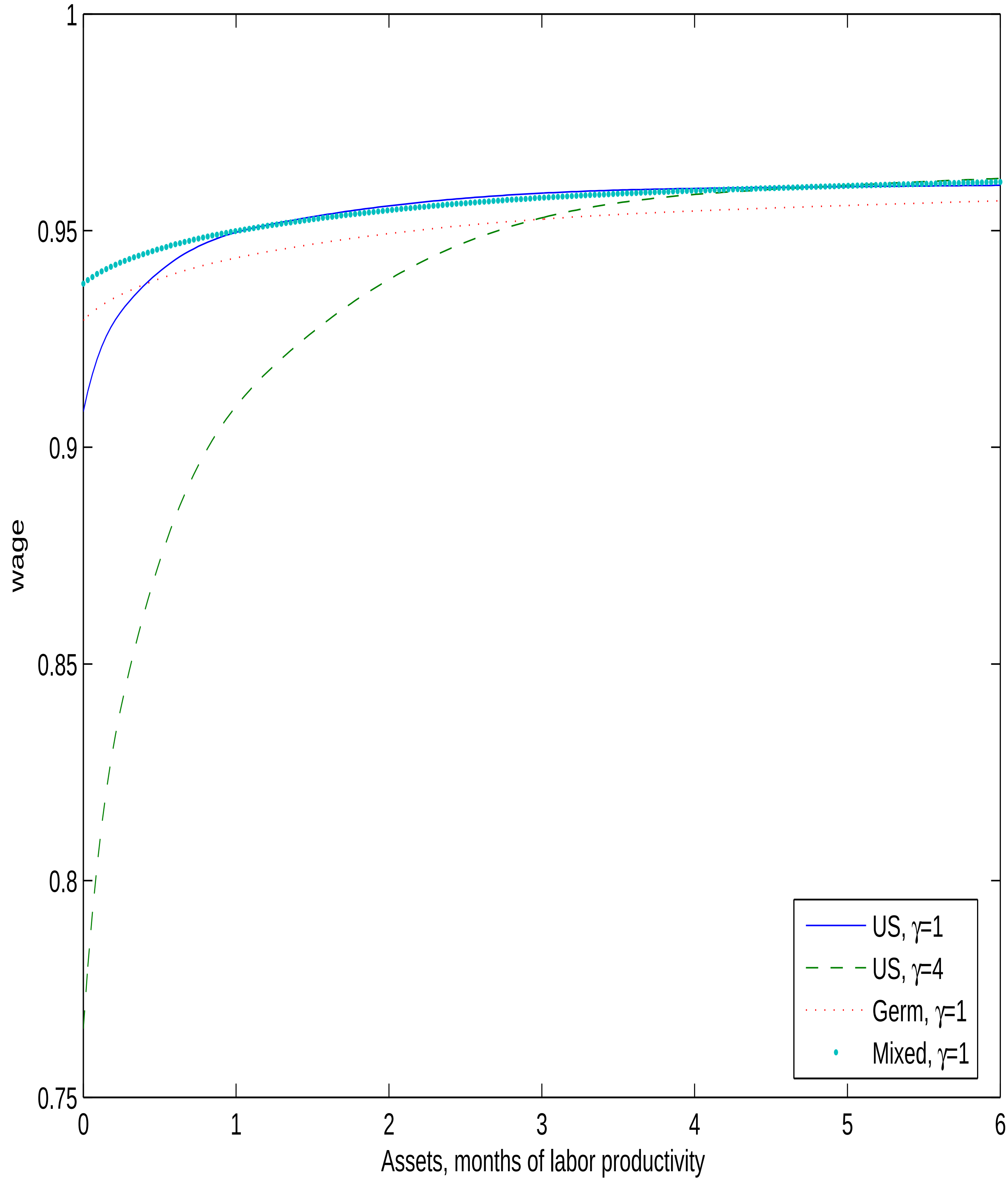

Figure 2: Wage as function of of workers' assets, US calibration

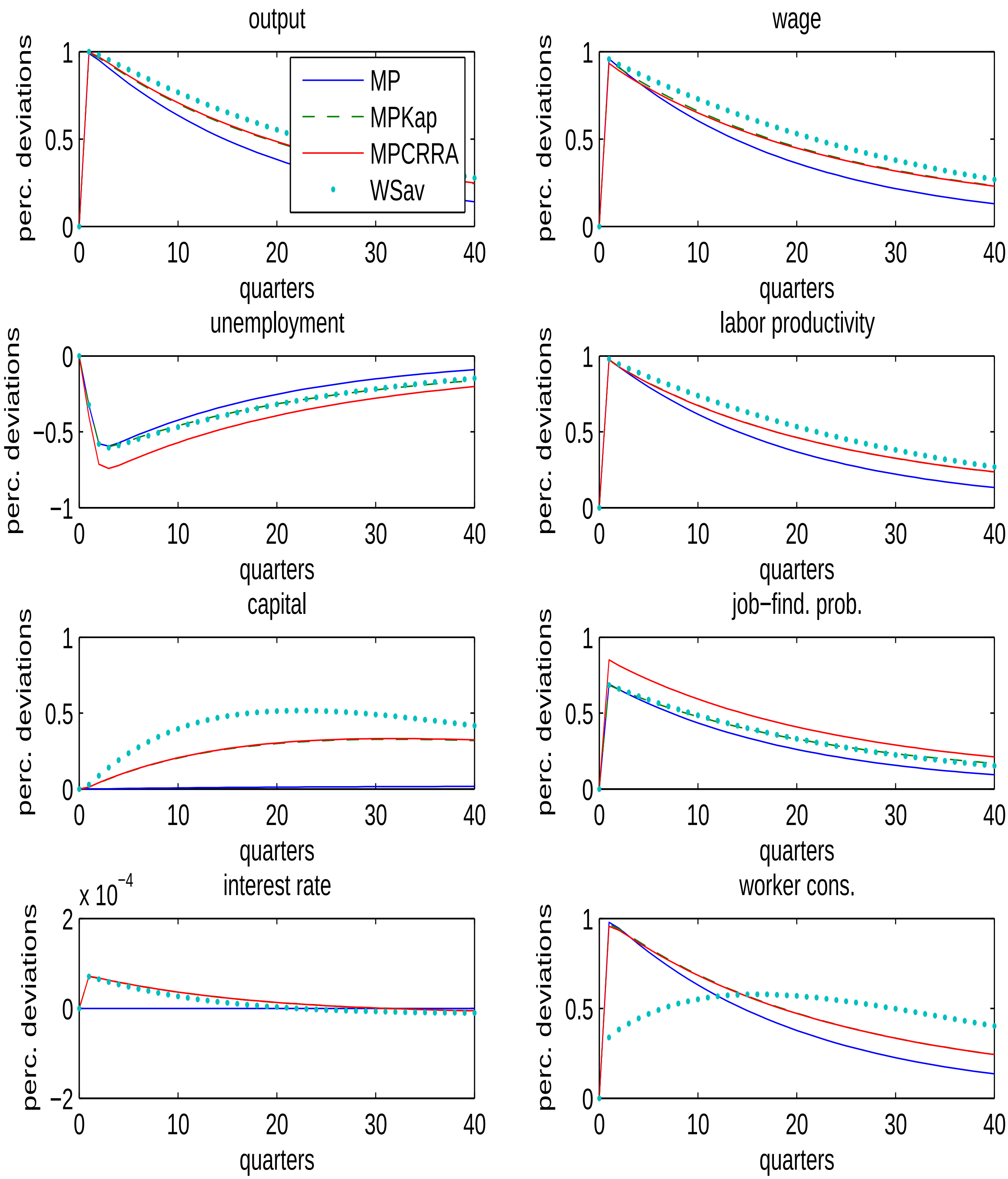

Figure 3: Reaction to productivity shock, US calibration, $\gamma = 1$

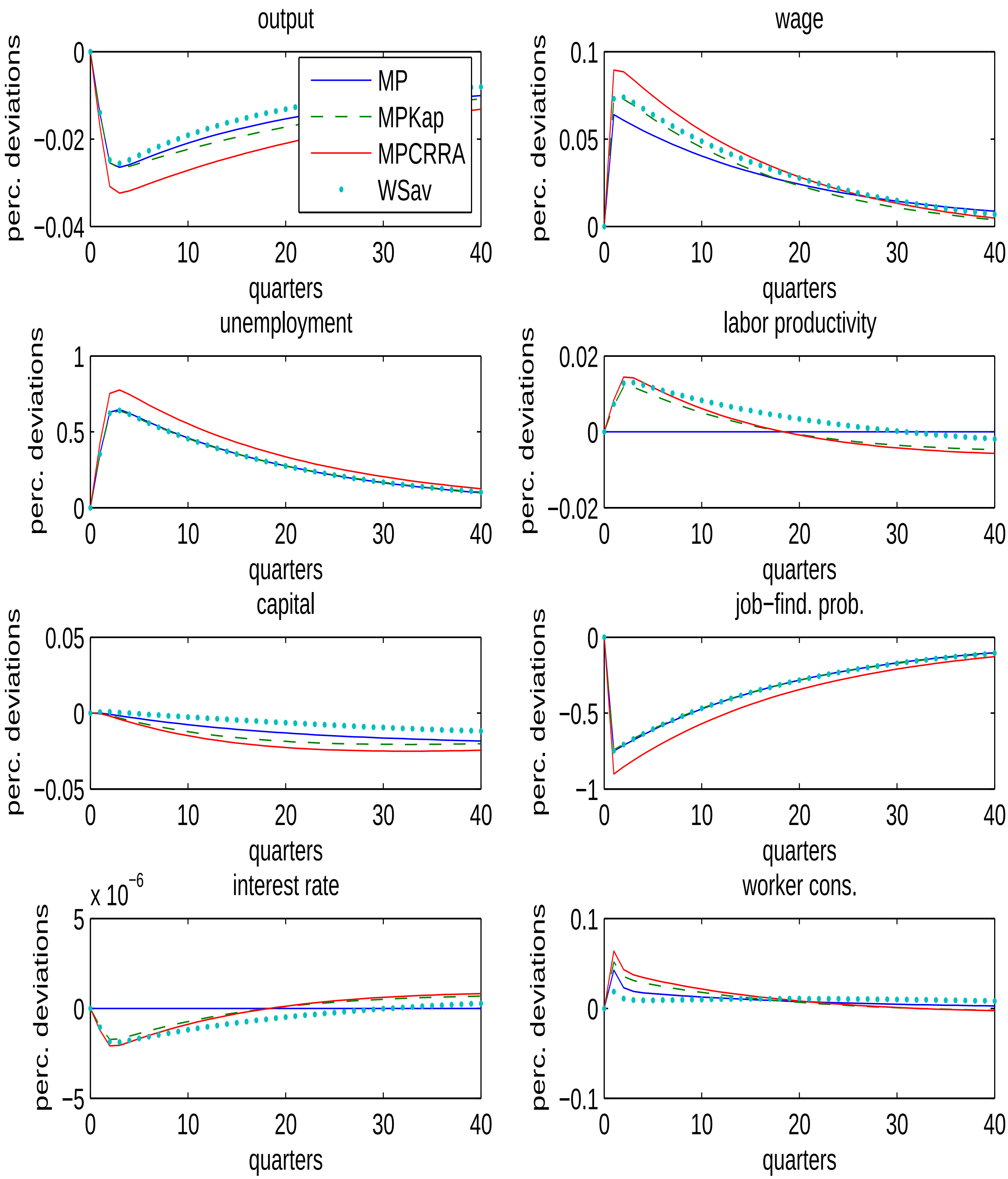

Figure 4: Reaction to bargaining shock, US calibration, $\gamma = 1$

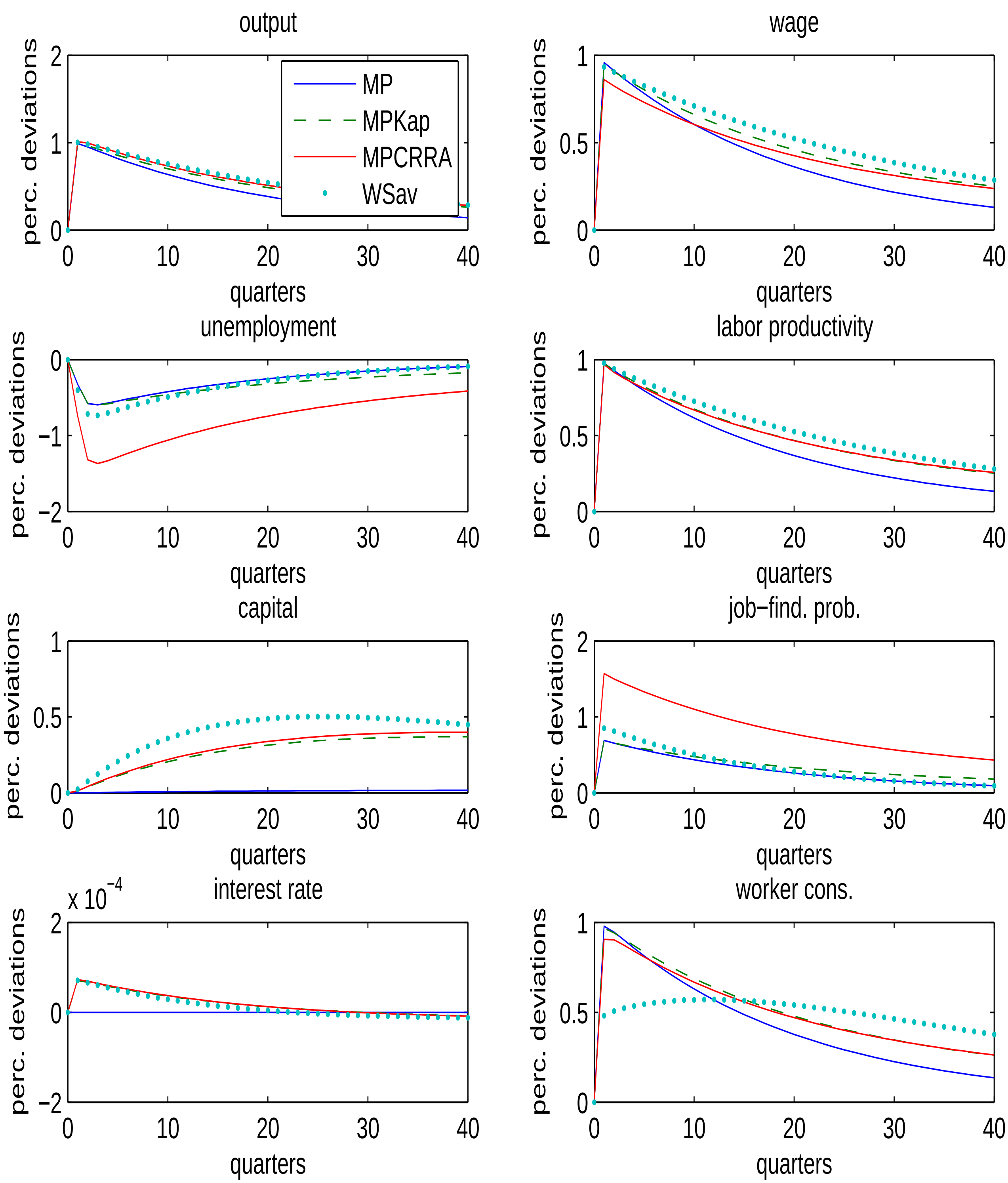

Figure 5: Reaction to productivity shock, US calibration, $\gamma = 4$

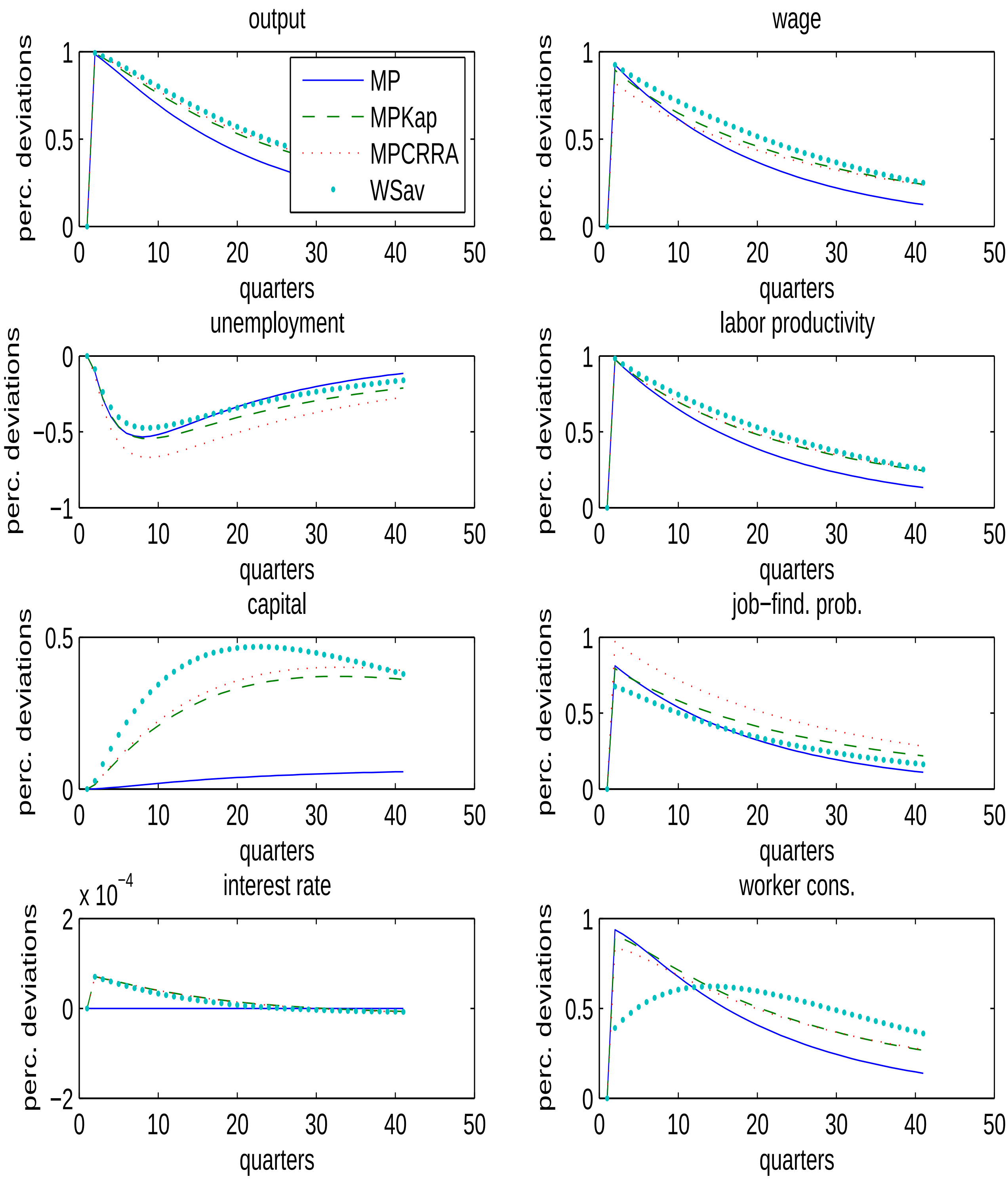

Figure 6: Reaction to productivity shock, high u - low k calibration, $\gamma = 1$

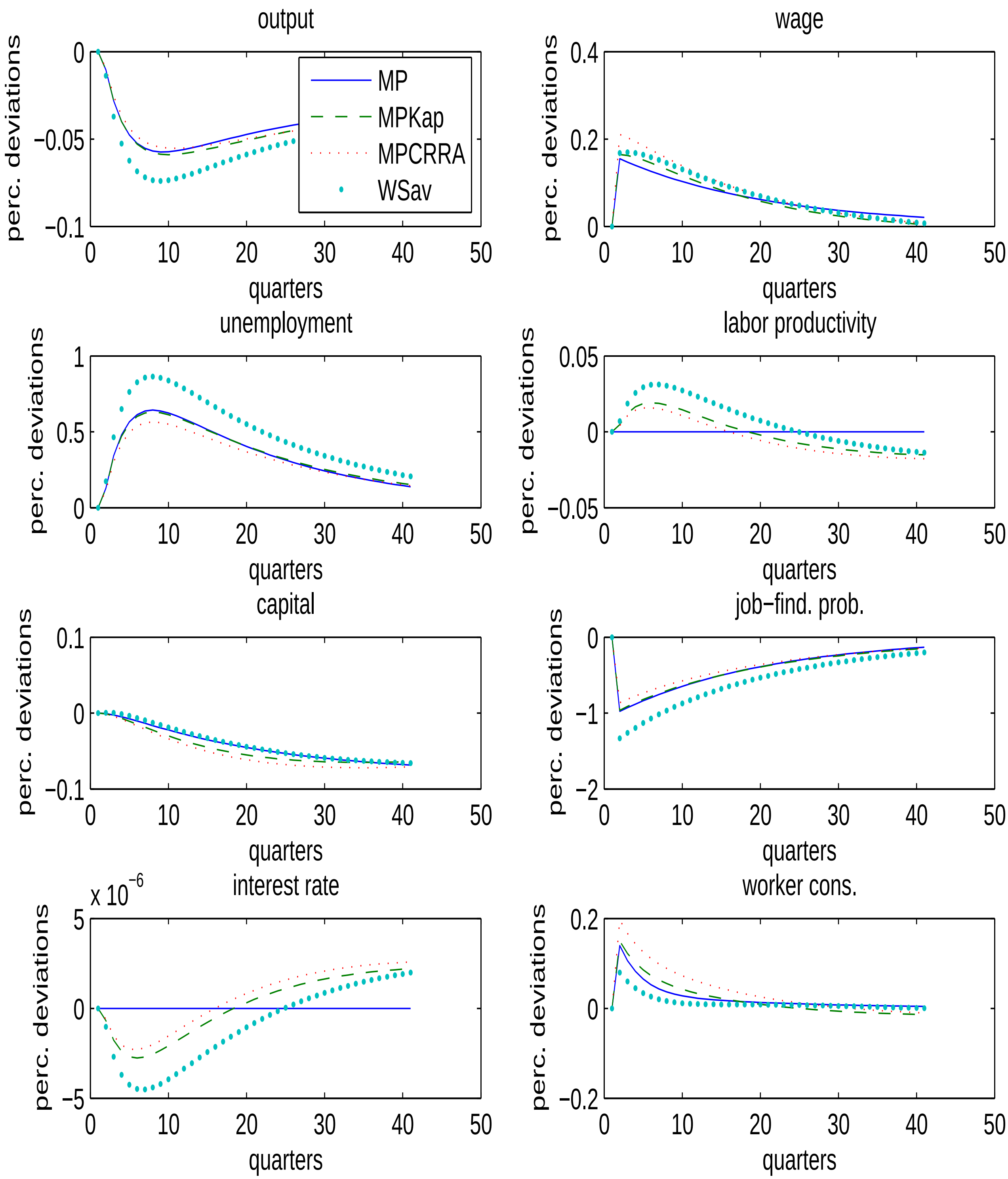

Figure 7: Reaction to bargaining shock, high u - low k calibration, $\gamma = 1$

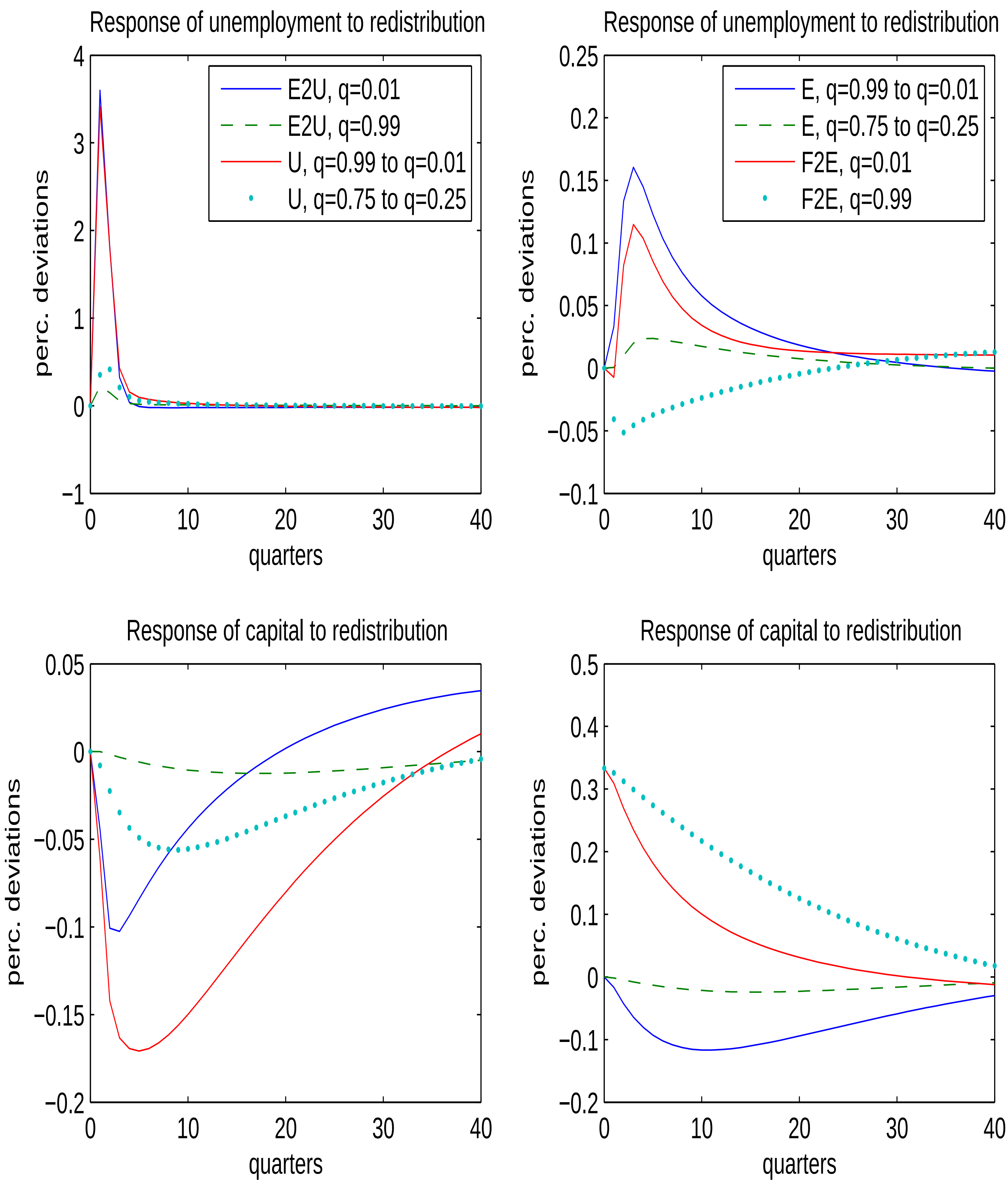

Figure 8: Transitions, US calibration, $\gamma = 1$

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A Distribution dynamics

To represent the cross-sectional distribution, we truncate the distribution of capital at a maximum level $\overline{k}$. The truncation point $\overline{k}$ is chosen such that in the steady state distribution, only a negligible fraction of households is close to $\overline{k}$. Then we split the support into n_d small intervals by a grid of points $\underline{k} = \kappa_0, \kappa_1, \ldots, \kappa_{n_d} = \overline{k}$. Then we characterize the cross-sectional distribution of wealth holdings by the employed workers by the mass at the lower bound, $\phi_t^e(0) \equiv \Phi_t^e(\underline{k})$, and the mass on the intervals $\phi_t^e(0) \equiv \Phi_t^e(\underline{k})$, and the mass on the intervals $\phi_t^e(i) \equiv \Phi_t^e(\kappa_i) - \Phi_t^e(\kappa_{i-1})$, $i = 1, \ldots, n_d$, which are stacked into the vector $\vec{\phi}_t^e \equiv (\phi_t^e(0), \phi_t^e(1), \ldots, \phi_t^e(n_d))$. For the unemployed, the probabilities $\phi_t^u(i)$ are defined in the same way.

Next we describe the dynamics of the probabilities $\phi_t^e(i)$ and $\phi_t^u(i)$ and their associated densities. We use the fact that the time interval of the model is very short (about 1 hour!), so that the saving that a household does within a period is much smaller than the width of the intervals (κ_{i-1}, κ_i). If saving is positive at κ_i, the flow of probability mass from i to $i+1$ is positive, since mass flows from interval (κ_{i-1}, κ_i) to (κ_i, κ_{i+1}). Otherwise it is negative, because mass flows from (κ_i, κ_{i+1}). to (κ_{i-1}, κ_i).

$$\Phi_t(i \to i+1; S) = \begin{cases} S\frac{\phi_t^s(i)}{\kappa_i - \kappa_{i-1}} & \text{if } S \geq 0 \\ S\frac{\phi_t^s(i+1)}{\kappa_{i+1} - \kappa_i} & \text{if } S < 0 \end{cases} \tag{30}$$

for $i = 1, \ldots, n_d$. Since $\phi_t^s(0)$ describes the probability mass at the point $\underline{k}$, we have the special case

$$\Phi_t(0 \to 1; S) = \begin{cases} \phi_t^s(0) & \text{if } S > 0 \\ \frac{k_c(\Omega - \underline{k})}{\kappa_1 - \underline{k}} & \text{if } S \leq 0 \end{cases} \tag{31}$$

The densities on the interval (κ_{i-1}, κ_i) is given by

$$f_t^s(i) \equiv \frac{\phi_t^s(i)}{\kappa_i - \kappa_{i-1}}, \qquad s \in \{e, u\} \tag{32}$$

The probabilities $\phi_t^e(i)$ and $\phi_t^u(i)$ follow the dynamic equations

$$\phi_{t+1}^e(i) = (1-\sigma)\phi_t^e(i) + p_t^W \phi_t^u(i) + S_t^e(\kappa_{i-1}) f_t^e(\mathcal{J}_1^e(2)) - S_t^e(\kappa_i) f_t^e(\mathcal{J}_1^e(2)) \tag{33}$$

$$\phi_{t+1}^u(i) = (1-p_t^W)\phi_t^u(i) + \sigma\phi_t^e(i) + S_t^u(\kappa_{i-1}) f_t^u(\mathcal{J}_1^u(2)) - S_t^u(\kappa_i) f_t^u(\mathcal{J}_1^u(2)) \tag{34}$$

where $\mathcal{J}_1^e(2)$ defines the "source interval" from where the savings flow comes:

$$\mathcal{J}_t^s(i) \equiv \begin{cases} i & \text{if } S_t^s(\kappa_i) \geq 0 \\ i+1 & \text{if } S_t^s(\kappa_i) < 0 \end{cases}, \qquad s \in \{e, u\} \tag{35}$$

If $S_t^s(\kappa_i) \geq 0$, probability mass is flowing from interval i to $i+1$, otherwise the probability flows from $i+1$ to i.

The probabilities $\phi_t^e(i)$ and $\phi_t^u(i)$ are defined at the end of period t. Since it turns out that, in equilibrium, employed workers who start with zero assets save a positive amount, at the end of the period no employed worker ends up at the borrowing constraint:

$$\phi_t^e(0) = 0 \tag{36}$$

Since unemployed workers with zero capital consume all their income, we get

$$\phi_{t+1}^u(0) = (1 - p_t^W)\left[\phi_t^u(0) \frac{k_c(\Omega - \underline{k})}{\kappa_1 - \underline{k}} \phi_t^u(1)\right] \tag{37}$$