Simple Model of Financial Frictions

Michael Reiter

Institute for Advanced Studies, Vienna

CEU, Macroeconomic Theory II, 2020

The Agency Problem in Corporate Finance

- We study the model of Tirole (2006, Sections 3.2 and 3.4).
- “The essence of corporate finance is that investors cannot appropriate the full benefit attached to the investment they enable.” [Tirole, p.116]
- The aim is to show that asymmetric information puts constraints on the financing of projects, even if those have positive NPV.

Entrepreneur

- has initial wealth A
- can shirk ($s = 1$) or not ($s = 0$)
- can run project of size I, which costs I and
  - gives revenues R with probability p_H (if $s = 0$) or p_L (if $s = 1$)
  - gives revenues 0 with probability $1 - p_H$ (if $s = 0$) or $1 - p_L$ (if $s = 1$)
- if project, is undertaken, has utility $U = y_E + sB$, where y_E is his income from the project and B is private benefit of shirking.
- if project is not undertaken, has utility $U = A$.
- if $I > A$, needs outside investors (lenders) to finance the difference $I - A$.

Assumptions

- Interest rate in the economy is 0 (to simplify notation)
- Lenders are competitive and risk neutral, get expected return 0
- Project is worthwhile if $s = 0$:

$$Rp_H > I \tag{1}$$

- Project is not worthwhile if $s = 1$:

$$Rp_L + B < I \tag{2}$$

- Effort is socially optimal if

$$B \leq R(p_H - p_L) \tag{3}$$

which we assume.

Optimal lending contract

- Since effort is unobservable, payments to investors can only depend on outcome of project, not on effort.
- We assume limited liability: if revenues are 0, nothing can be paid to entrepreneur or lenders: $y_{E,0} = y_{L,0} = 0$.
- To make effort optimal, income of entrepreneur in case of success, $y_{E,1}$, must be big enough:

$$(p_H y_{E,1} + (1 - p_H) y_{E,0}) \geq (p_L y_{E,1} + (1 - p_L) y_{E,0}) + B \tag{4}$$

  which is equivalent (because of $y_{E,0} = 0$) to

$$y_{E,1} \geq \frac{B}{(p_H - p_L)} \tag{5}$$

- Then income of lenders is constrained by

$$y_{L,1} \leq R - \frac{B}{(p_H - p_L)} \tag{6}$$

Can the project be financed?

- The maximum expected gross return to outside investors is

$$p_H \frac{R - \frac{B}{(p_H - p_L)}}{I - A} \tag{7}$$

- This must not be smaller than 1, therefore

$$A \geq I - p_H R + p_H \frac{B}{(p_H - p_L)} \tag{8}$$

- The entrepreneur needs positive assets if

$$p_H R - I \leq p_H \frac{B}{(p_H - p_L)} \tag{9}$$

Notice that the NPV condition (1) requires $p_H R - I \geq 0$.

Interpretation

- If the entrepreneur has no assets, her reservation utility is zero. In the absence of agency problems, the project would be undertaken if it gives the normal return to lenders, and zero to the entrepreneur.
- If the project is realized, the entrepreneur can obtain private benefits B from shirking. To prevent this, she must be incentivized by earning more in case of success, namely (cf. Equ. (4))

$$y_{E,1} - y_{E,0} \geq \frac{B}{(p_H - p_L)} \tag{10}$$

- Because of limited liability, there is a bound on $y_{E,0}$. Here we have assumed $y_{E,0} \geq 0$. This puts a lower bound on expected earnings of entrepreneurs, namely

$$p_H y_{E,1} \geq \frac{p_H}{(p_H - p_L)} B \tag{11}$$

Quantitative importance of the friction

Equ. (11) would allow $p_H y_{E,1}$ to be arbitrarily big. However, Assumptions (1) and (2) imply that

$$R(p_H - p_L) > B \tag{12}$$

Putting this into (11) the required $p_H y_{E,1}$ cannot be larger than $p_H R$. This can still be almost all the revenues from the project!

Remark on effort and shirking

The benefit of shirking B only arises if the project is undertaken.
In contrast, the benefit of not undertaking entrepreneurial effort would also arise if the project is not undertaken (at least this is the most natural interpretation).
Formulating the model in terms of shirking rather than effort therefore makes the project worthwhile under a larger set of parameter values.
In the effort formulation, the NPV condition (1) is $I - p_H R \leq -B$. Then it is harder to satisfy NPV and require a positive A, cf. condition (9).

Main Lesson

Whenever there is unobservable effort, an entrepreneur (manager, banker) must be given sufficiently big stake in the project.

$\Longrightarrow$

Not all the returns of an investment project can be pledged to outside investors.

$\Longrightarrow$

If the entrepreneur has not sufficient own equity, a project with positive NPF may not be undertaken because of lack of financing.

Continuous Investment

- Investment $I \in [0, \infty)$.
- Income
  1. $R \cdot I$ in case of success
  2. 0 in case of failure
- Misbehaving yields private benefits $B \cdot I$, reduces probability of success from p_H to p_L.
- $p_H R > 1$
- $p_L R + B < 1$
- $p_H R - 1 < \frac{p_H B}{p_H - p_L}$; net revenue smaller than agency cost

Borrowing capacity

- Incentive compatibility:

$$y_{E,1} \geq \frac{BI}{p_H - p_L} \tag{13}$$

- Break-even condition:

$$p_H(RI - y_{E,1}) \geq I - A \tag{14}$$

- Combining (13) and (14) gives

$$I \leq \frac{1}{1 - p_H(R - B/(p_H - p_L))} A \tag{15}$$

- Competitive lenders $\Longrightarrow$ borrower's net utility is

$$U_E^{net} = (p_H R - 1)I \tag{16}$$

Therefore: invest as much as possible.

Shadow value of equity

$$U_E = U_E^{net} + A = \frac{p_H R - 1}{1 - p_H(R - B/(p_H - p_L))} A + A$$
$$= \frac{p_H B/(p_H - p_L)}{1 - p_H(R - B/(p_H - p_L))} A \tag{17}$$

Shadow value of equity is

$$\frac{p_H B/(p_H - p_L)}{p_H B/(p_H - p_L) - (p_H R - 1)} > 1 \tag{18}$$

Debt vs. Equity

- If the project generates zero income in case of failure, debt and equity are equivalent. Zero return can result from
  1. all equity,
  2. debt that defaults because of no project return
- If there is positive income even in case of failure of the project,
  - all income should go to the investor, to provide the strongest incentives to the entrepreneur (borrower).
  - this means that the finance is not all equity, otherwise the entrepreneur would also get a share of the profit
  - the optimum is to issue debt such that all income in the case of failure accrues to the investor.

Different ways of modeling the financial friction

- Only a part of the return of the project can be “pledged” to outside investors.
- Entrepreneurs can only issue riskless debt.
- Debt must be collateralized (cannot exceed certain fraction of value of collateral).
- Costly state verification (if return of project is not sufficient to pay back debt, low return is verified at a cost).

Tirole, J. (2006).
The Theory of Corporate Finance.
Princeton University Press.